

# The Trouton Experiment and $E = mc^2$

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In the Fall of 1900, Frederick T. Trouton started work on an ingenious experiment in his laboratory at Trinity College in Dublin. The purpose of the experiment was to detect the earth's presumed motion through the ether, the 19th century medium thought to carry light waves and electric and magnetic fields. The experiment was unusual in that, unlike most of these so-called ether drift experiments, it was not an experiment in optics. Trouton tried to detect ether drift by charging and discharging a capacitor in a torsion pendulum at its resonance frequency, which he hoped would set the system oscillating.

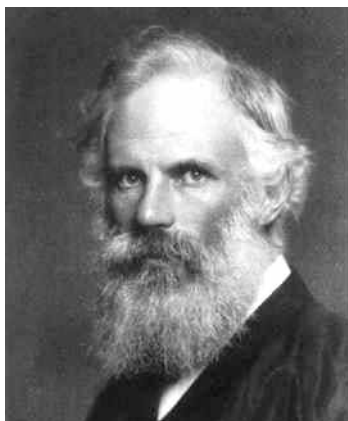


Fig. 1. FitzGerald

The basic idea behind the experiment came from George Francis FitzGerald (see Fig. 1), whose assistant Trouton was at the time. Consider Fig. 2 below. A battery is used to charge a capacitor. If the power is switched on, an electromagnetic field is produced largely confined to the volume between the plates of the capacitor. If the system is at rest in the ether, the charges will only produce an electric field; if the system is moving, the charges will also produce a magnetic field. As Trouton wrote in his paper on the experiment:

The question then naturally arises as to the source supplying the energy required to produce this magnetic field. If we attribute it to the electric generator, say a battery, there is no difficulty [...] FitzGerald's view, however, was that it would be found to be supplied through there being a mechanical drag on the condenser itself at the moment of charging (Trouton 1902, 557–558)

In other words, FitzGerald thought that the energy for the magnetic field would come from the capacitor's kinetic energy. The capacitor's kinetic energy is proportional to the square of its velocity ( $E_{\text{kin}} = (1/2)mv^2$ ), so if it loses some kinetic energy, it must lose some of its velocity as well. In other words, if FitzGerald were right, a moving capacitor upon being charged should experience a jolt in the direction opposite to its direction of motion.

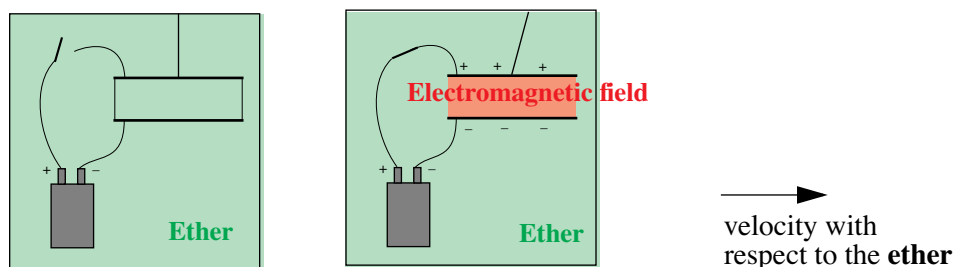
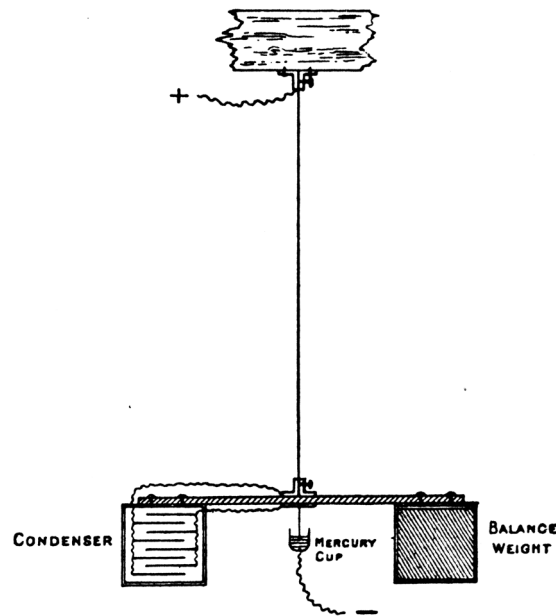


Fig. 2. A capacitor and a battery moving through the ether

In Fig. 3, the effect is illustrated for a capacitor suspended on a wire from the ceiling of the laboratory with its plates parallel to the direction of motion. The actual arrangement, shown in Fig. 3 taken from Trouton's paper, was a little more subtle. At FitzGerald's suggestion, Trouton made

1. This handout is based on Janssen 2002a, 2002b.

the capacitor part of a torsion pendulum. The capacitor was “charged and discharged continuously by means of a clock-work, at the intervals corresponding to the free period of swing of the apparatus. In this way any effect produced would cumulate and be made easier of observation” (Trouton 1902, 560).<sup>1</sup>



**Fig. 3.** Trouton’s torsion pendulum with capacitor (Trouton 1902, 560)

FitzGerald died in February 1901 before the experiment was concluded. It was thus left to others to try and reconcile Trouton’s result with then current electromagnetic theory. The first to do so was Joseph Larmor, who not only got closely involved with Trouton’s experiment after FitzGerald’s death, but who also became the editor of a volume of FitzGerald’s scientific papers published the following year. Trouton’s paper on the experiment suggested by FitzGerald was reprinted in this volume accompanied by a four-page editorial note (Larmor 1902). Larmor, however, devoted only one short paragraph of his note to Trouton’s original experiment, confidently asserting that no effect should have been expected in the first place.

Larmor argued as follows. Suppose the capacitor is held fixed in the laboratory. According to FitzGerald the energy for the magnetic field then would have to come from a tiny decrease in the kinetic energy of the earth as a whole. But this would violate the so-called center-of-mass theorem, according to which nothing happening inside an isolated system can change the state of motion of the center of mass of the system.<sup>2</sup> One might object that the earth-capacitor system is not fully isolated because it interacts with the ether. However, since Larmor assumed the ether to

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1. There is an obvious improvement of Trouton’s design. As Trouton explains in his paper: “It was originally intended to have two condensers, one at each end of the cross arm, the one to be charged at the moment the other was discharged, not only to double the effect, but also to secure a pure torque acting on the wire. This idea had to be abandoned in the final experiment, owing to all the condensers available breaking down under the excessive voltage employed save only one” (Trouton 1902, 559).
  2. Here is an example of an application of the center-of-mass theorem: Imagine yourself standing on ice on a pair of skates holding a heavy brick. If you throw the brick away from you in the forward direction, you yourself will start moving backwards. The net motion of you and the brick remains zero.

be immobile, the center-of-mass theorem is also violated if we consider the ether-earth-capacitor system. Larmor's simple rebuttal of FitzGerald's suggestion thus seems perfectly adequate.

The problem is that it was unclear whether the center-of-mass theorem actually holds in a theory based on an immobile ether. The center-of-mass theorem is closely related to Newton's third law, the principle that action equals reaction, which, in turn, is closely related to the law of momentum conservation.<sup>1</sup> When Larmor wrote his comment on the Trouton experiment, the status of momentum conservation in its various guises in theories positing an immobile ether had been the subject of some serious debate, notably between the Dutch physicist H.A. Lorentz and the French mathematician Henri Poincaré. In 1902, the situation was unclear at best.

Newton's principle of the equality of action and reaction is hard to reconcile with the notion of an ether that can set matter in motion (through the Lorentz forces of electromagnetic fields on charged particles), yet can itself never be set in motion by matter. Lorentz clearly stated this obvious difficulty in a widely read monograph of 1895. After discussing the problem of how to make sense of forces acting on an immobile ether and concluding that the easiest way to solve the problem would be never to apply the notion of force to the ether at all, Lorentz wrote, in an often quoted passage:

It is true that this conception would violate the principle of the equality of action and reaction—because we do have grounds for saying that the ether exerts forces on ponderable matter—but nothing, as far as I can see, forces us to elevate that principle to the rank of a fundamental law of unlimited validity. (Lorentz 1895, 28; italics in the original)

Poincaré strongly objected to this aspect of Lorentz's theory, especially to the violations of the center of mass theorem it entails. In fact, he made this the topic of his contribution to a *Festschrift* on the occasion of the 25th anniversary of Lorentz's doctorate (Poincaré 1900).

Poincaré illustrated his objection with the example of a mirror recoiling upon the reflection of light (Poincaré 1900b, 273). He used this same example in an important lecture during the International Congress of Arts and Sciences in St. Louis in 1904:

Imagine, for example, a Hertzian oscillator, like those used in wireless telegraphy; it sends out energy in every direction; but we can provide it with a parabolic mirror, as Hertz did with his smallest oscillators, so as to send all the energy produced in a single direction. What happens then according to the theory? The apparatus recoils, as if it were a cannon and the projected energy a ball; and that is contrary to the [action equals reaction] principle of Newton, since our projectile here has no mass, it is not matter, it is energy (Poincaré 1904, 101; my italics)

The italicized final remark, which is not to be found in Poincaré's more detailed discussion of the example in 1900, shows how tantalizingly close he came to the resolution of the problem through  $E = mc^2$ .

In a letter to Poincaré in response to the latter's contribution to his *Festschrift*, Lorentz reiterated that any theory based on an immobile ether will violate the action equals reaction principle and thereby the center of mass theorem. He made it clear that he did not see this as a serious problem for his theory.

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1. The same example given in the preceding footnote can also be used to illustrate "action = - reaction" and momentum conservation. In accordance with the conservation of momentum, the total momentum before and after throwing the brick is zero. After you have thrown the brick, the brick's momentum,  $m_{\text{brick}}v_{\text{brick}}$ , and your momentum,  $m_{\text{you}}v_{\text{you}}$ , are equal and opposite and will add up to zero. Note that your velocity will be smaller than the brick's velocity because your mass is greater than the brick's mass and the product of mass and velocity needs to be the same for both of you.

From the point of view of classical mechanics, Poincaré's recoiling mirror example also violates momentum conservation. This can be avoided by ascribing momentum to the electromagnetic field. The concept of electromagnetic momentum was introduced by the German theoretical physicist Max Abraham in 1903 (Abraham 1903). Today physicists are so accustomed to a concept of momentum that is broader than mechanical momentum that it is easy to forget that this was by no means obvious at the beginning of the century. This point is made very nicely in the following passage from a paper by Planck. The paper is based on a lecture delivered during the annual *Congress of German Natural Scientists and Physicians (Versammlung Deutscher Naturforscher und Ärzte)* in Cologne on September 23, 1908. Planck's paper, entitled "Comments on the Principle of Action and Reaction in General Dynamics," contains a vivid description of the difficulties surrounding the action equals reaction principle around the turn of the century:

As is well-known, the real content of the Newtonian principle of the equality of action and reaction is the theorem of the constancy of the quantity of motion or of the momentum of motion; I therefore want to talk about this principle only in the sense of that theorem, and, more specifically, about its relevance for general dynamics, which not only includes mechanics in a more restricted sense, but also electro-dynamics and thermodynamics.

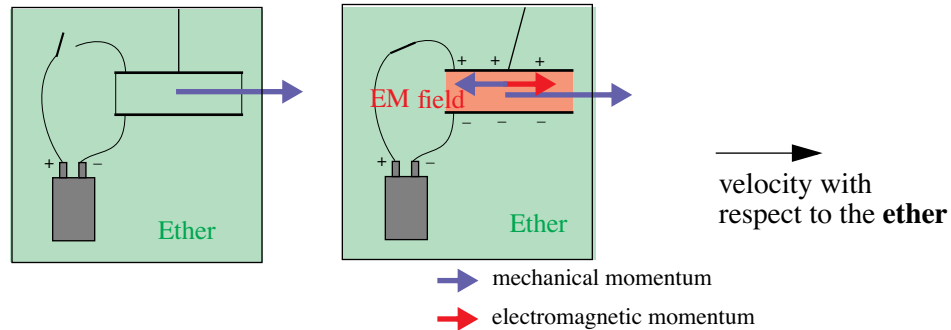
Many of us will still recall the stir it caused, when H. A. Lorentz, in laying the foundations of an atomistic electrodynamics on the basis of a stationary ether, denied Newton's third axiom absolute validity, and inevitably this circumstance was turned into a serious objection against Lorentz's theory, as was done, for instance, by H. Poincaré. A calmness of sorts [eine Art Beruhigung] only returned when it became clear, especially through the investigations of M. Abraham, that the reaction principle could be saved after all, in its full generality at that, if only one introduces, besides the mechanical quantity of motion, the only kind known at that point, a new quantity of motion, the electromagnetic kind. Abraham made this notion even more plausible by a comparison between the conservation of the quantity of motion and the conservation of energy. Just as the energy principle is violated if one does not take electromagnetic energy into account and satisfied if one does introduce this form of energy, so is the reaction principle violated if one only considers the mechanical quantity of motion but satisfied as soon as one also takes into account the electromagnetic quantity of motion.

However, this comparison, incontestable in and of itself, leaves one essential difference untouched. In the case of energy, we already knew a whole series of different kinds—kinetic energy, gravitation [sic], elastic energy of deformation, heat, chemical energy—so it does not constitute a fundamental innovation if one adds electromagnetic energy to these different forms of energy as yet another form. In the case of the quantity of motion, however, we only knew one kind so far: the mechanical kind. Whereas energy was already a universal physical concept, the quantity of motion had so far been a typically mechanical concept and the reaction principle had been a typically mechanical theorem. Consequently, its generalization, while recognized to be necessary, was bound to be experienced as a revolution of a fundamental nature, through which the up to that point relatively simple and uniform concept of the quantity of motion acquired a considerably more complicated character. (Planck 1908, 828–829)

Planck may have exaggerated the difficulties physicists were experiencing with the notion of electromagnetic momentum somewhat for rhetorical purposes (he goes on to show that the idea of putting energy and momentum on equal footing is a very natural one in relativity theory), but this passage does make it clear that the introduction of electromagnetic momentum was indeed, as Planck says, a "fundamental innovation."

Electromagnetic momentum plays a central role in Lorentz's analysis of the Trouton experiment (Lorentz 1904). Lorentz's reasoning is illustrated in Fig. 4. When the moving capacitor is charged, electromagnetic momentum is created pointing in the direction of motion. For the total momentum to be conserved, this gain in electromagnetic momentum must be compensated by a loss of mechanical momentum of the capacitor. This means that the capacitor's velocity must

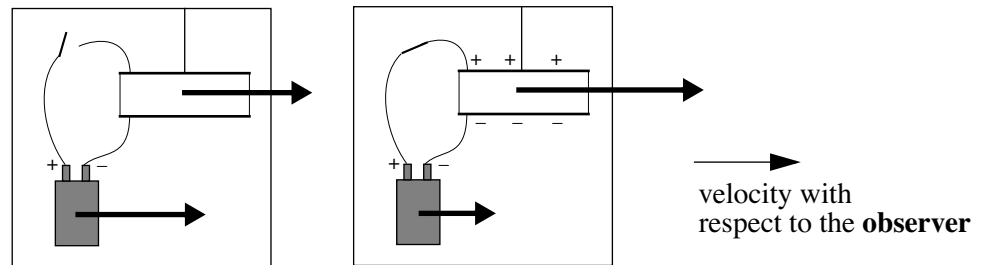
decrease (recall that momentum is the product of mass and velocity,  $p = mv$ ). In other words, Lorentz agreed with FitzGerald that if a moving capacitor is charged, it should experience a sudden jolt. As he wrote: “This momentum being produced at the moment of charging and disappearing at that of discharging, the condenser must experience in the first case an impulse  $[-p]$  and at the second an impulse  $[+p]$ . However Trouton has not been able to observe these jerks. I believe it may be shown ... that the sensibility of the apparatus was far from sufficient for the object Trouton had in view” (Lorentz 1904, 829–830). So, Lorentz thought that in principle the effect predicted by FitzGerald should occur but that Trouton’s experiment had not been sensitive enough to detect it.



**Fig. 4.** Lorentz’s analysis of the Trouton experiment

Notice the peculiar situation we have found. Larmor argued: *if the effect predicted by FitzGerald does occur, the center-of-mass theorem is violated*. Lorentz argued: *if the effect does not occur, conservation of momentum is violated*. It seems that we are facing a dilemma. We need to make a choice between momentum conservation and the center-of-mass theorem, two laws that are essentially equivalent in Newtonian mechanics.

This is where the most famous equation of all of modern physics,  $E = mc^2$ , comes to the rescue. We can easily wiggle out of our dilemma once we realize that energy has mass. The argument runs as follows. If energy has mass, a transfer of energy from the battery to the capacitor means a transfer of mass, and, in a frame of reference in which battery and capacitor are moving, a transfer of momentum. So, Fig. 4, showing the momentum of the capacitor in the Trouton experiment before and after it is charged, should be replaced by Fig. 5 below, showing the momentum of both the capacitor and the battery before and after the capacitor is charged.

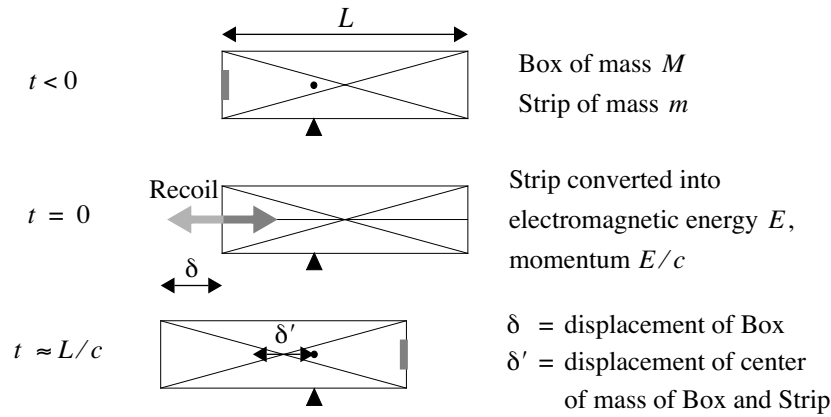


**Fig. 5.** Transfer of momentum in the Trouton experiment

When the moving capacitor is charged, it gains a certain amount of energy, mass, and momentum, while the moving battery loses that same amount of energy, mass, and momentum. The total amount of momentum is conserved. Contrary to what Lorentz thought in 1904, this does not

require the capacitor to change its velocity. The increase in the capacitor's momentum,  $\Delta \mathbf{p}$ , corresponds to a change in the capacitor's mass,  $\Delta m = \Delta E / c^2$ , not to a change in its velocity:  $\Delta \mathbf{v} = 0$ . Hence, there is no violation of the center of mass theorem. Once the inertia of energy is taken into account, a strictly negative result of the Trouton experiment is thus seen to be compatible both with momentum conservation and with the center of mass theorem.  $E = mc^2$  has saved the day.

About a year after he first introduced the inertia of energy, Einstein published a paper, entitled "The Principle of the Conservation of Motion of the Center of Gravity and the Inertia of Energy," in which he showed that  $E = mc^2$  is necessary and sufficient to ensure that the center of mass theorem holds for systems in which "not only mechanical, but also electromagnetic processes take place" (Einstein 1906, 627). As Einstein acknowledges, his paper is similar to Poincaré's contribution to the Lorentz *Festschrift* (Poincaré 1900). Einstein showed that in order to avoid the kind of violations of the center of mass theorem discussed by Poincaré, one has to assume that energy has inertia. Instead of Poincaré's recoiling mirror, Einstein considered the thought experiment illustrated in Fig. 6.



**Fig. 6.** Einstein's thought experiment to show that  $E = mc^2$  is necessary and sufficient for the center of mass theorem

Consider a box of mass  $M$  and length  $L$ . Suppose some energy  $E$  is stored on the inside of the left wall of the box, and suppose that at time  $t = 0$  this energy is somehow converted into electromagnetic radiation travelling to the other side of the box. The radiation is absorbed at the other end of the box, where the energy is converted back to its original form. According to standard electromagnetic theory, the box will recoil upon emission of the radiation, and it will recoil again upon re-absorption of the radiation, bringing the box back to rest. Standard electromagnetic theory tells us that the radiation will have momentum ( $E/c$ ). Momentum conservation requires that the box will recoil with that same momentum in the opposite direction. So, what this thought experiment shows is that by moving energy from one side of the box to the other, the completely isolated system of box plus energy  $E$  can move itself. If the energy  $E$  has no mass, this is in blatant violation of the center of mass theorem. With the help of fig. 6, it can easily be shown that the only way to avoid this consequence is to ascribe mass  $m = E/c^2$  to the energy  $E$ .

Let the energy  $E$  initially be contained in a strip of as yet unknown mass  $m$  much smaller than the mass  $M$  of the box ( $m \ll M$ ). The strip is stuck against the inside of the left wall of the box. This means that the center of mass of box plus strip will be slightly to the left of the middle of the box. The energy is then converted into electromagnetic radiation and, a short time later, recon-

verted into a strip of mass  $m$  stuck against the inside of the right wall of the box. The center of mass of box plus strip is now slightly to the right of the middle of the box. The displacement  $\delta'$  of the center of mass can be calculated from the following condition that determines where a wedge supporting the system should be placed so that the system is perfectly balanced:

$$(1) \quad M(\delta'/2) = m((L - \delta')/2).$$

It follows that, to a very good approximation, the displacement of the center of mass is given by:

$$(2) \quad \delta' \approx (m/M)L.$$

The center of mass theorem is satisfied if and only if the displacement  $\delta'$  of the center of mass to the right is equal to the distance  $\delta$  that the box travels to the left during the time it takes for the radiation to move from one end of the box to the other. To a very good approximation, the time that the box is in flight can be set equal to  $(L/c)$ , and the velocity of the box can be set equal to its momentum  $E/c$  divided by its mass  $M$ . Hence, to a very good approximation, the distance travelled by the box is given by:

$$(3) \quad \delta \approx ([E/c^2]/M)L.$$

Comparing eqs. (2) and (3), one sees that indeed

$$(4) \quad \delta = \delta' \Leftrightarrow E = mc^2.$$

The conclusion is that  $E = mc^2$  is the necessary and sufficient condition for the center of mass theorem to hold in systems in which processes involving both electromagnetic fields and ordinary matter occur. Strictly speaking, there should of course be approximately-equal signs rather than equal signs in eq. (4), just as in eqs. (2) and (3). In other words, the thought experiment only yields the conclusion to a very good approximation. Einstein was happy to leave it at that (Einstein 1906, 629).

The Trouton experiment can be seen as a practical version of Einstein's thought experiment. In the case of the Trouton experiment it is the conversion of chemical energy of the battery into the energy of the electromagnetic field between the plates of the capacitor that would lead to a violation of the center of mass theorem were it not for the inertia of energy expressed in  $E = mc^2$ . Unfortunately, the Trouton experiment, rather than living on in physics textbooks as a beautiful illustration of its most famous equation, has been completely and thoroughly forgotten.

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