1 Thermal diffusion

The following statement can be found in a guidebook: "The surface temperature of the Mediterranean sea varies between 12 degrees (Celsius) in winter and 26 degrees in the summer. However, at a depth of 40 meters, the temperature is constant"

1) In the model discussed in the lectures, can you use the surface of the sea as the plane $z = 0$? If not, why?

2) Assuming the model can be used from a depth of 10 meters, and knowing that the thermal conductivity of sea water at 20degrees is $\lambda = 0.6$ J.m^{-1}.s^{-1}.K^{-1}, and that the heat capacity per unit volume is $C = 4.2\ 10^6$J.K^{-1}.m^{-3}, discuss the accuracy of the statement contained in the guidebook. You will numerically estimate the amplitude of the temperature variation at a depth of 40m. Specify the hypothesis made concerning the changes of λ and T with temperature.

2 Hydrodynamics

Consider the flow of a liquid between two parallel plates, driven by a pressure difference $(P_1 - P_0)$ between points at coordinates $x = 0$ and $x = L$ (Poiseuille flow). The plates are parallel to Oxy, and the flow is in the x direction. The fluid has a shear viscosity η, a mass density ρ. The distance between the plates is H (corresponding to the planes $z = 0$ and $z = H$.

1) Recall the computation of the flow profile when the boundary condition on the plates is one of "no slip" (vanishing velocity on the solid plates). Compute the mass flow rate as a function of the pressure gradient.

Make a schematic plot of the flow profile.

2) The boundary condition on the solid plates is now assumed to be a "partial slip" condition, which in this situation is written as:

$$\frac{\partial v_x}{\partial z} = +\frac{1}{\delta} v_x \quad \text{at} \quad z = 0$$

$$\frac{\partial v_x}{\partial z} = -\frac{1}{\delta} v_x \quad \text{at} \quad z = H$$

δ is a length that characterizes the boundary and is claled the "slip length". Solve the flow problem with these new boundary conditions. First, show that the flow still has a parabolic profile. Next, apply the boundary conditions to determine the integration constants.

3) With these new boundary conditions, compute the flow rate as a function of the pressure gradient. Plot the velocity profile and give a physical interpretation of the "slip length" δ.

3 Elasticity

Consider a solid elastic sphere of radius R. This sphere is rotating at an angular velocity Ω. Its mass density is ρ and its Young's modulus is E.

The inertia force can be associated with a local force density $\rho r \Omega^2$.

Use dimensional analysis (and the hypothesis of linear elasticity) to estimate (up to some unknown numerical factor) the relative increase in the radius of the sphere, $\delta R/R$ associated with the rotation.

Apply this result to the earth, using numerical values for E,R,Ω,ρ that seem reasonable to you.