From directed polymer to interface growth

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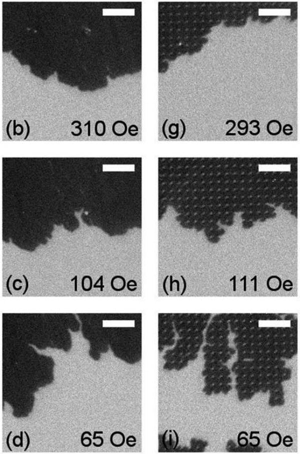
CPTGA – Grenoble – 24 May 2017

1D interfaces

Interfaces in magnetic films



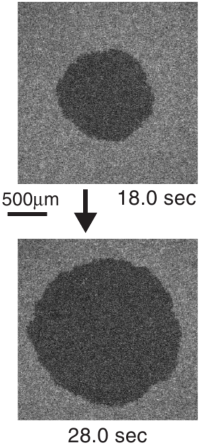
from Metaxas *et al.*

APL **94** 132504 (2009)

Large range of physical scales

Wide spectrum of phenomena

Growth in liquid crystals



from Takeuchi & Sano

PRL **104** 230601 (2010)

Disordered elastic systems

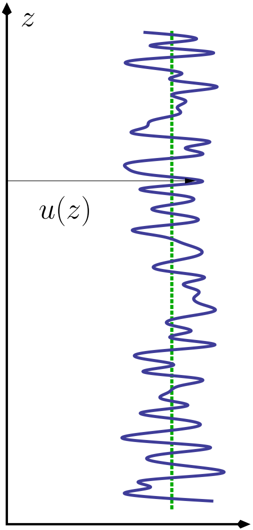
- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\text{el}} = \frac{c}{2}\int dz\,\big(\nabla u(z)\big)^2$$ **[Short-range]**

$$\mathcal{H}^{\text{el}} = \frac{c}{2\pi}\int dz dz'\,\frac{\big(u(z)-u(z')\big)^2}{(z-z')^2}$$ [Long-range]

- Disorder: tends to bend it

$$\mathcal{H}_V^{\text{dis}} = \int dz\, V\big(z,u(z)\big)$$



Competition btw “order” and “disorder”

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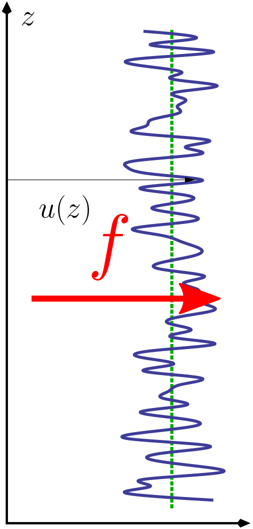
- Force: induces motion of the interface



Competition btw "order" and "disorder"

Some known results

1. Huge variety of physical systems and theoretical approaches
   - ⋆ Elastic manifolds (lines, membranes, interfaces); periodic (vortex lattices); growth interfaces (aggregation, wetting)
   - ⋆ Methods: field theory, renormalisation group [L. Canet, M. Tarpin,...], scaling analysis, integrable models [E. Ragoucy,...], exactly solvable models [V. Beffara, E. Bertin,...], replica methods
   - ⋆ Reviews: Halpin-Healy&Zhang; Blatter&*al.*; Quastel; Corwin
2. Nature of fluctuations in **dimension 1+1** (elastic line)
   - ⋆ No disorder ($V(z,u) \equiv 0$): *diffusive* ($u \sim z^{1/2}$), **Edwards-Wilkinson** (EW)
   - ⋆ Disorder ($V(z,u) \not\equiv 0$): *super-diffusive* ($u \sim z^{2/3}$), **Kardar-Parisi-Zhang** (KPZ)

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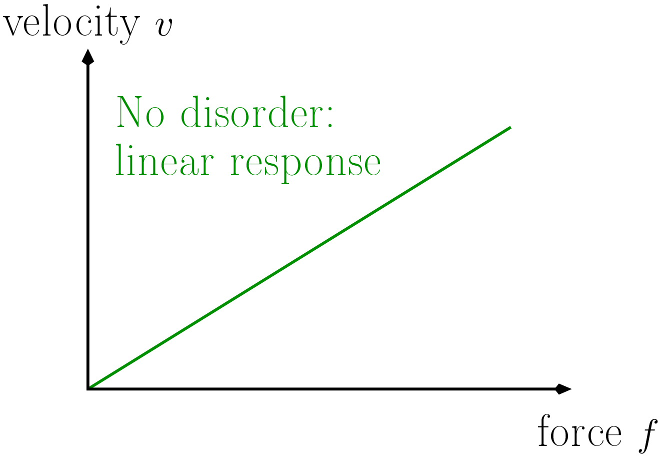
Depinning transition @ no disorder

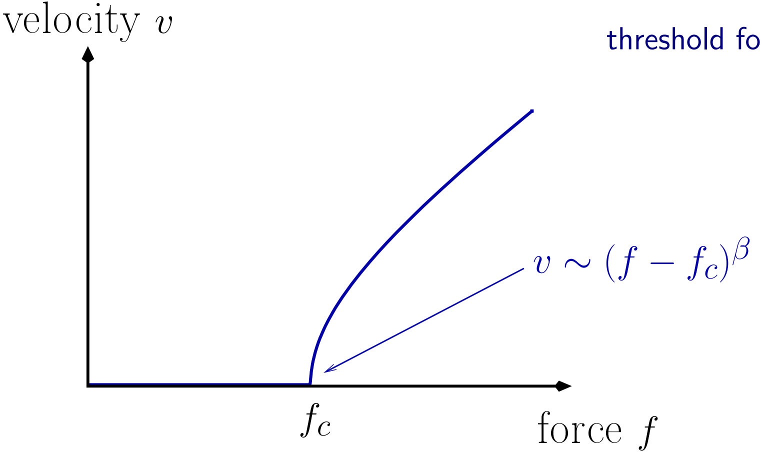
3. Non-equilibrium dynamics:



Depinning transition @ zero temperature

3. Non-equilibrium dynamics: depinning

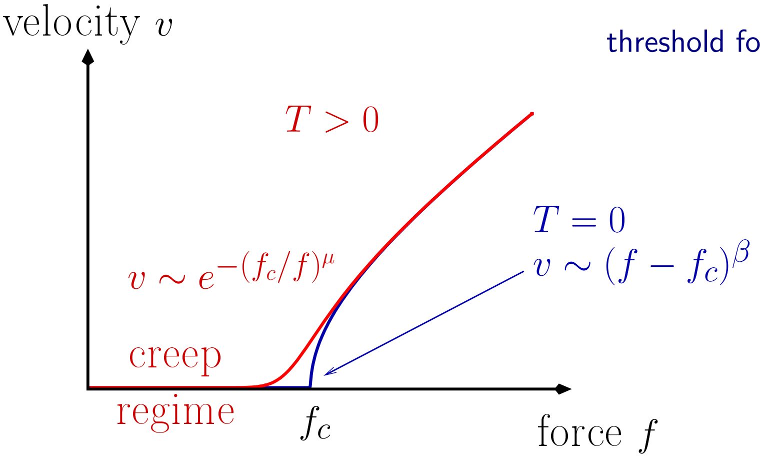
threshold force f_c



Depinning transition @ finite temperature

3. Non-equilibrium dynamics: depinning, creep

threshold force f_c



Outline

Study of 1D models

1. Equilibrium properties $(f = 0)$
   Scaling exponents of the rough geometry
   Identification of relevant lengthscales
   [work with Elisabeth Agoritsas, Thierry Giamarchi]

2. Non-equilibrium $(f \ll f_c)$
   Creep law
   Effective description
   [work with Reinaldo García-García, Elisabeth Agoritsas, Lev Truskinovsky and Damien Vandembroucq]

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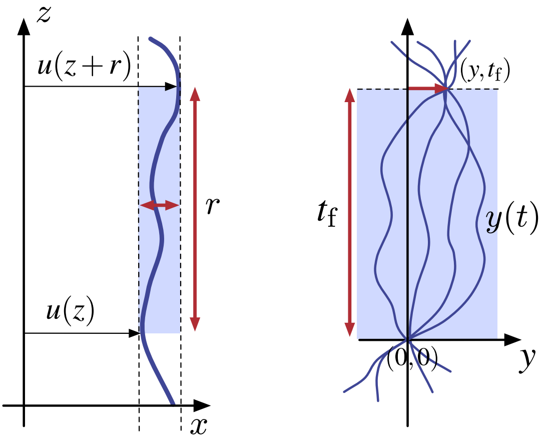
Role of short-range correlated disorder.

1D interface in the Directed Polymer (DP) language

- No bubbles
- No overhangs
- Interface lengthscale r

$\updownarrow$

DP 'time' t_f



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integration of fluctuations at scales smaller than t_f

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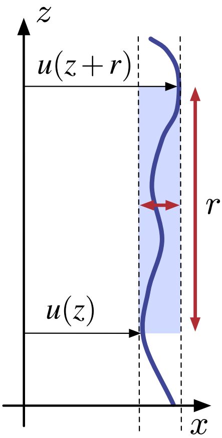

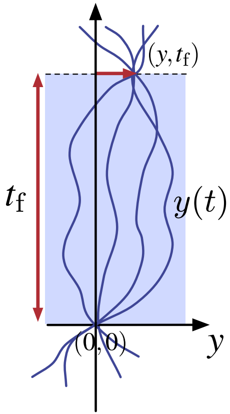

working at fixed 'time' t_f $\iff$
integration of fluctuations at scales smaller than t_f

lengthscale $\equiv$ time duration

Equilibrium directed polymer at temperature T (and $f=0$)

- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\mathsf{el}}[y(t),t_{\mathrm{f}}] = \frac{c}{2}\int_0^{t_{\mathrm{f}}} dt\, \big[\partial_t y(t)\big]^2$$

- Disorder: tends to bend it

$$\mathcal{H}_V^{\mathsf{dis}}[y(t),t_{\mathrm{f}}] = \int_0^{t_{\mathrm{f}}} dt\, V\big(t,y(t)\big)$$

Competition btw "order" and "disorder"

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} weight of $(y(t))_{0<t<t_{\mathrm{f}}}$:

$$\mathrm{e}^{-\mathcal{H}_V / T}$$

with

$$\mathcal{H}_V = \mathcal{H}^{\mathsf{el}} + \mathcal{H}_V^{\mathsf{dis}}$$

Competition btw "order" and "disorder"

- Interpretations of the elastic Hamiltonian:

elasticity or kinetic energy or Wiener measure

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Competition btw "order" and "disorder"

- Ingredients up to now:

elastic constant c disorder potential $V(t, y)$ temperature T

A naive scaling argument 1/3

Roughness function (variance of the end-point fluctuations):

$$B(t_{\rm f}) = \overline{\langle y(t_{\rm f})^2 \rangle} \qquad \overline{\,\cdot\,} = \text{disorder average} \qquad \langle\,\cdot\,\rangle = \text{thermal average}$$

Path-integral writing:

$$B(t_{\rm f}) = \int \mathcal{D}V \;\mathbb{P}[V\,]\, \frac{\int_{y(0)=0} \mathcal{D}y(t)\, y(t_{\rm f})^2\, \mathrm{e}^{-\frac{1}{T}\mathcal{H}[y(t),V;t_{\rm f}]}}{\int_{y(0)=0} \mathcal{D}y(t)\, \mathrm{e}^{-\frac{1}{T}\mathcal{H}[y(t),V;t_{\rm f}]}}$$

Uncorrelated disorder: Gaussian, centered, with

$$\overline{V(t,y)V(t',y')} = D\,\delta(t'-t)\delta(y'-y)$$

which rescales as

$$V(b\,\hat{t}, a\,\hat{y}) \overset{d}{=} a^{-\frac{1}{2}} b^{-\frac{1}{2}} D^{\frac{1}{2}} \hat{V}(\hat{t},\hat{y})$$

A naive scaling argument 2/3

Flory rescaling: $t = t_{\mathrm{f}}\,\hat{t}, \qquad y = t_{\mathrm{f}}^{\zeta_{\mathrm{F}}}\left(\frac{D}{c^2}\right)^{\frac{1}{5}}\hat{y}, \qquad \zeta_{\mathrm{F}} = \frac{3}{5}$
ensures

$$\frac{1}{T}\mathcal{H}^{\mathsf{el}} = \frac{(cD^2)^{1/5}}{T} t_{\mathrm{f}}^{1/5}\ \frac{1}{2}\int_0^1 d\hat{t}\,\left[\partial_{\hat{t}}\hat{y}(\hat{t})\right]^2$$

$$\frac{1}{T}\mathcal{H}^{\mathsf{dis}} \overset{d}{=} \frac{(cD^2)^{1/5}}{T} t_{\mathrm{f}}^{1/5}\ \int_0^1 d\hat{t}\,\hat{V}\big(\hat{t},\hat{y}(\hat{t})\big)$$

with correlations:

$$\overline{\hat{V}(\hat{t},\hat{y})\hat{V}(\hat{t}',\hat{y}')} = \delta(\hat{t}'-\hat{t})\delta(\hat{y}'-\hat{y})$$

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Rescaling of the roughness

$$B(t_\mathrm{f}) \;=\; \left[\frac{D}{c^2}\right]^{2/5} t_\mathrm{f}^{6/5}\, \hat{B}(t_\mathrm{f})$$

$$\hat{B}(t_\mathrm{f}) \;=\; \overline{\frac{\displaystyle\int_{\hat{y}(0)=0} \mathcal{D}\hat{y}(\hat{t})\; \hat{y}(1)^2 \exp\left\{-\frac{(cD^2)^{\frac{1}{5}}}{T} t_\mathrm{f}^{\frac{1}{5}} \int_0^1 d\hat{t}\, \left[\tfrac{1}{2}(\partial_{\hat{t}}\hat{y})^2 + \hat{V}(\hat{t},\hat{y}(\hat{t}))\right]\right\}}{\displaystyle\int_{\hat{y}(0)=0} \mathcal{D}\hat{y}(\hat{t}) \exp\left\{-\frac{(cD^2)^{\frac{1}{5}}}{T} t_\mathrm{f}^{\frac{1}{5}} \int_0^1 d\hat{t}\, \left[\tfrac{1}{2}(\partial_{\hat{t}}\hat{y})^2 + \hat{V}(\hat{t},\hat{y}(\hat{t}))\right]\right\}}}$$

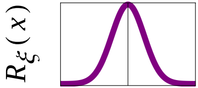
If saddle-point trajectory $\hat{y}^\star(\hat{t})$ exists at $t_\mathrm{f} \to \infty$, **it is indep. of** t_f

$$B(t_\mathrm{f}) \;=\; \left[\frac{D}{c^2}\right]^{2/5} t_\mathrm{f}^{6/5}\, \overline{\hat{y}^\star(1)^2}$$ not the expected KPZ behaviour $\sim t_\mathrm{f}^{4/3}$

A naive scaling argument 3/3

Where is $t_{\rm f}$? In the disorder correlations on a lengthscale ξ:

$$\overline{V(z,x)V(z',x')} = D\,\delta(z'-z)R_\xi(x'-x)$$



scaling as $\quad R_\xi(x) = \frac{1}{\xi}\,R_{\xi=1}(x/\xi)$

A naive scaling argument 3/3

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{ the $t_\mathrm{f}\to\infty$ saddle-point trajectory
$\hat{y}^\star(\hat{t})$ **depends on t_f through $\hat{\xi}(t_\mathrm{f})$** }

A naive scaling argument 3/3

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$$B(t_{\mathrm{f}}) \;=\; \left[\tfrac{D}{c^2}\right]^{2/5} t_{\mathrm{f}}^{6/5}\,\overline{\hat{y}^\star(1)^2} \qquad \Big\{\cdots\Big\} \Rightarrow \text{modifies the exponent } \frac{6}{5}\mapsto\frac{4}{3}$$

Free-energy fluctuations of trajectories $(0,0) \rightsquigarrow (t,y)$

How to solve this issue?

- Partition function Z_V *vs.* Free-energy F_V

$$Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t')\mathrm{e}^{-\frac{1}{T}\mathcal{H}_V[y(t'),t]} \qquad F_V(t,y) = -\frac{1}{T}\log Z_V(t,y)$$

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- **Stochastic Heat Equation** (Feynman-Kac formula)

$$\partial_t Z_V = \Big[\frac{T}{2c}\partial_y^2 - \frac{1}{T}V(t,y)\Big] Z_V(t,y) \qquad \textbf{(SHE)}$$

  Linear, multiplicative noise, reversible

- **Kardar-Parisi-Zhang** equation

$$\partial_t F_V = \frac{T}{2c}\partial_y^2 F_V - \frac{1}{2c}\big[\partial_y F_V\big]^2 + V(t,y) \qquad \textbf{(KPZ)}$$

  Non-linear, additive noise, non-reversible

  $F_V(t,y) \equiv$ interface height at position y and time t

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  Linear, multiplicative noise, $Z_V(0,y) = \delta(y)$

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  Non-linear, additive noise, $F_V(0,y)$: "sharp wedge" initial cond.
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Statistical tilt symmetry

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- **Statistical Tilt Symmetry**

$$F_V(t,y) = \underbrace{c\frac{y^2}{2t} + \frac{T}{2}\log\frac{2\pi T t}{c}}_{\text{thermal contribution } F_{V\equiv 0}} + \underbrace{\bar{F}_V(t,y)}_{\text{disorder contribution}} \qquad \textbf{(STS)}$$

- **Tilted** KPZ equation for $\bar{F}_V(t,y)$

$$\partial_t \bar{F}_V + \frac{y}{t}\partial_y \bar{F}_V = \frac{T}{2c}\partial_y^2 \bar{F}_V - \frac{1}{2c}\left[\partial_y \bar{F}_V\right]^2 + V(t,y)$$

Non-linear, additive noise, $\bar{F}_V(0,y) \equiv 0$: "simple" initial cond.

Known results @$\xi = 0$ $[\Longleftrightarrow T \to \infty$ @$\xi > 0]$

- **Infinite-time limit** $t_{\mathrm{f}} \to \infty$ (steady state)

  $\bar{F}(t_{\mathrm{f}} = \infty, y)$ distributed as a 2-sided Brownian Motion

  i.e.: $\mathbb{P}\big[\bar{F}(t_{\mathrm{f}} = \infty, y)\big]$ Gaussian, of correlator

$$\overline{\big[\bar{F}(t_{\mathrm{f}} = \infty, y) - \bar{F}(t_{\mathrm{f}} = \infty, y')\big]^2} = \widetilde{D}\,|y - y'| \quad \text{with} \quad \boxed{\widetilde{D} = \frac{cD}{T}}$$

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- Rescaling of the disorder free-energy $\bar{F}(t_{\mathrm{f}} = \infty, a\hat{y})$

$$\bar{F}(t_{\mathrm{f}} = \infty, a\hat{y}) \;\overset{d}{=}\; a^{1/2}\widetilde{D}^{1/2} \underbrace{\hat{F}(t_{\mathrm{f}} = \infty, \hat{y})}_{\widetilde{D}=1}$$

Saddle-point argument for the KPZ exponent

Flory rescaling for the free-energy

$$t = t_{\mathrm{f}}\hat{t}, \qquad y = (\widetilde{D}/c^2)^{\frac{1}{3}}\, t_{\mathrm{f}}^{\frac{2}{3}}\, \hat{y}, \qquad \bar{F}_V(t,y) \overset{(d)}{=} (\widetilde{D}^2 t_{\mathrm{f}}/c)^{1/3} \hat{F}(\hat{t},\hat{y})$$

Rescaling of the roughness

$$B(t_{\mathrm{f}}) \underset{t_{\mathrm{f}}\to\infty}{\sim} \left[\frac{\widetilde{D}}{c^2}\right]^{\frac{2}{3}} t_{\mathrm{f}}^{\frac{4}{3}}\; \hat{B}(t_{\mathrm{f}})$$

$$\hat{B}(t_{\mathrm{f}}) \;=\; \overline{\frac{\displaystyle\int_{\mathbb{R}} d\hat{y}\, \hat{y}^2 \exp\Big\{-\tfrac{1}{T}\big(\tfrac{\widetilde{D}^2}{c}t_{\mathrm{f}}\big)^{\frac{1}{3}}\Big[\tfrac{\hat{y}^2}{2} + \hat{F}(\hat{t},\hat{y})\Big]\Big\}}{\displaystyle\int_{\mathbb{R}} d\hat{y}\, \exp\Big\{-\tfrac{1}{T}\big(\tfrac{\widetilde{D}^2}{c}t_{\mathrm{f}}\big)^{\frac{1}{3}}\Big[\tfrac{\hat{y}^2}{2} + \hat{F}(\hat{t},\hat{y})\Big]\Big\}}}$$

The $t_{\mathrm{f}} \to \infty$ saddle-point $[\hat{B}(t_{\mathrm{f}}) \sim \overline{(\hat{y}^\star)^2} \sim t_{\mathrm{f}}^0]$ gives the correct exponent

Extensions and open questions

- Control of $\xi > 0$ and $t_{\mathrm{f}} < \infty$ (where $\bar{F}$ is not Brownian): difficult and hard problem.
- How not to use the Brownian steady state?
- Replica trick

$$\overline{\langle \mathcal{O}\left[y(t_{\mathrm{f}})\right]\rangle} = \int \mathcal{D}V\,\mathbb{P}\left[V\right] \frac{\int_{y(0)=0} \mathcal{D}y(t)\,\mathcal{O}\left[y(t_{\mathrm{f}})\right]\,\mathrm{e}^{-\frac{1}{T}\mathcal{H}[y(t),V;t_{\mathrm{f}}]}}{\int_{y(0)=0} \mathcal{D}y(t)\,\mathrm{e}^{-\frac{1}{T}\mathcal{H}[y(t),V;t_{\mathrm{f}}]}}$$

$$= \lim_{n\to 0} \int_{y_1(0)=0} \mathcal{D}y_1(t)\,\ldots\,\mathcal{D}y_n(t)\,\mathcal{O}\left[y_1(t_{\mathrm{f}})\right]\,\mathrm{e}^{-\frac{1}{T}\widetilde{\mathcal{H}}[y_1(t),\ldots,y_n(t);t_{\mathrm{f}}]}$$

mapping to interacting bosons

$$\widetilde{\mathcal{H}}\left[y_1(t),\ldots,y_n(t);t_{\mathrm{f}}\right] = \int_0^{t_{\mathrm{f}}} dt\left[\frac{c}{2}\sum_{a=1}^{n}\left(\partial_t y_a(t)\right)^2 - \frac{D}{T}\sum_{a,b=1}^{n} R_\xi\left(y_a(t)-y_b(t)\right)\right]$$

Extensions and open questions

- Control of $\xi > 0$ and $t_\mathrm{f} < \infty$ (where $\bar{F}$ is not Brownian): difficult and hard problem.
- How not to use the Brownian steady state?
- Replica trick [note to the initiated: $n \to 0$ limit not always required]

$$\overline{\langle \mathcal{O}\left[y(t_\mathrm{f})\right]\rangle} = \int \mathcal{D}V\, \mathbb{P}\left[V\right] \frac{\int_{y(0)=0} \mathcal{D}y(t)\, \mathcal{O}\left[y(t_\mathrm{f})\right]\, \mathrm{e}^{-\frac{1}{T}\mathcal{H}[y(t),V;t_\mathrm{f}]}}{\int_{y(0)=0} \mathcal{D}y(t)\, \mathrm{e}^{-\frac{1}{T}\mathcal{H}[y(t),V;t_\mathrm{f}]}}$$

$$= \lim_{n\to 0} \int_{y_1(0)=0} \mathcal{D}y_1(t) \ldots \mathcal{D}y_n(t)\, \mathcal{O}\left[y_1(t_\mathrm{f})\right]\, \mathrm{e}^{-\frac{1}{T}\widetilde{\mathcal{H}}[y_1(t),\ldots,y_n(t);t_\mathrm{f}]}$$

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  - ⋆ at $\xi = 0$: enormous amount of results (integrable models) [Le Doussal, Dotsenko, Schehr, Corwin...] Airy$_{1,2,\ldots}$, Tracy-Widow distributions

Extensions and open questions

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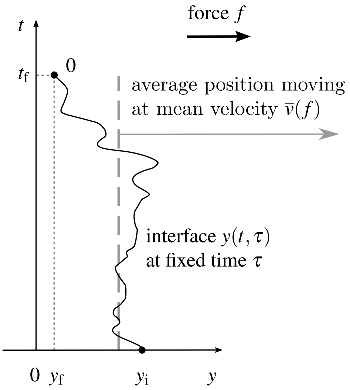
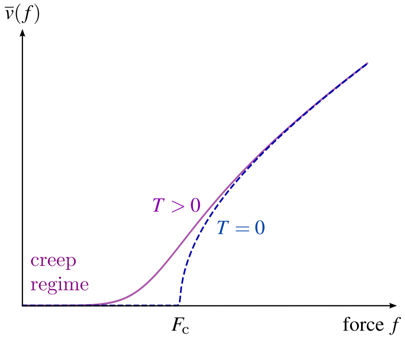
    [Le Doussal, Dotsenko, Schehr, Corwin...]
    Airy$_{1,2,\ldots}$, Tracy-Widow distributions
  - ⋆ at $\xi > 0$: few attempts
    · Bec, Khanin: Burgers turbulence
    · L. Canet, S. Mathey, EA, T. Kloss, VL: non-perturbative RG
    · E Agoritsas, VL:
    Gaussian variational method ($\equiv$ Hartree-Fock)
    Keeping $\xi > 0$ and $t_{\rm f} < \infty$ gives the correct exponent
    Non-trivial modifications of the Replica Symmetry Breaking
    (due to all scales mixed in the equation for self-energy)

Non-linear response at small force

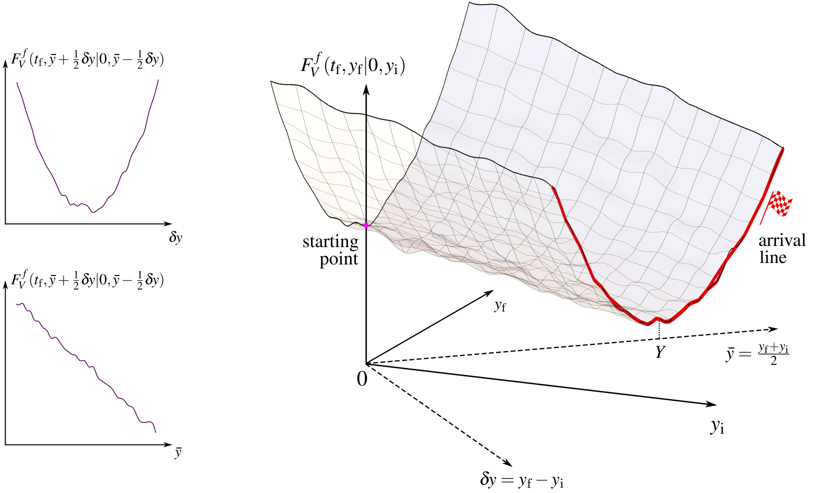




Creep law: non-linear response to small force

$$\text{velocity} \sim \exp\Big\{ - \Big[\overbrace{\frac{\text{critical force}}{\text{force}}}^{\text{depends on } c,\ D,\ T,\ \xi} \Big]^{1/4} \Big\}$$

Effective model



Mean velocity ↔ Mean First Passage Time problem (MFPT)

Effective model

Mean velocity $\leftrightarrow$ Mean First Passage Time problem (MFPT)

- Effective model at fixed t_{f}: quasistatic dynamics
  - ⋆ motion of an segment of length t_{f}
  - ⋆ effective description: motion of extremities follow Langevin dynamics
  - ⋆ forces derive from $F_V^f(t_{\mathrm{f}}, y_{\mathrm{f}} | t_{\mathrm{i}}, y_{\mathrm{i}})$
  - ⋆ exact at $f = 0$
- Optimization over t_{f} at fixed f
  - ⋆ optimal t_{f} yielding the avalanche size at fixed f
  - ⋆ saddle-point argument after rescaling
  - ⋆ yields the creep law

$$\text{velocity} \sim \exp\Big\{ - \Big[\frac{\text{critical force}}{\text{force}}\Big]^{1/4} \Big\}$$

  - ⋆ creep exponent $\frac{1}{4}$ related to the KPZ exponent $\frac{2}{3}$

Summary & open questions

- **Geometry** of interface $\longleftrightarrow$ Directed Polym. **free-energy** fluctuat.
  - ⋆ power-counting arguments can be tricky
  - ⋆ integrate fluctuations up to a large scale t_{f}
  - ⋆ ξ **plays a role at all lengthscales**
- **Dynamics** of interface
  - ⋆ non-linear response: creep law
  - ⋆ effective quasi-static picture

Summary & open questions

- **Geometry** of interface $\longleftrightarrow$ Directed Polym. **free-energy** fluctuat.
  - ⋆ power-counting arguments can be tricky
  - ⋆ integrate fluctuations up to a large scale $t_{\rm f}$
  - ⋆ ξ **plays a role at all lengthscales**
- **Dynamics** of interface
  - ⋆ non-linear response: creep law
  - ⋆ effective quasi-static picture
- Perspectives:
  - ⋆ experimental probe of the importance of ξ
  - ⋆ interpretation in other 'incarnations' of the KPZ class
    - . growth interfaces with $F(t,y) =$ height at (real) time t
    - . through replicæ: **1D quantum bosons** with softened repulsive interaction
  - ⋆ depinning transition?
  - ⋆ change of geometry (roughness exponent) as force increases?
  - ⋆ higher dimensions? Gaussian variational method

References

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  Elisabeth Agoritsas, VL
  J. Phys. A **50** 104001 (2017)
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- *Static fluctuations of a thick one-dimensional interface in the 1+1 directed polymer formulation*
  Elisabeth Agoritsas, VL and Thierry Giamarchi
  Phys. Rev. E **87** 042406 (2013)

Thank you for your attention! Bon appétit