Rare events: large deviations, numerical methods and phase transitions

Cécile Appert-Rolland, Juan P. Garrahan, Estelle Pitard,
Kristina van Duijvendijk, Frédéric van Wijland

Julien Tailleur, Jorge Kurchan

Thierry Bodineau, Bernard Derrida

Freddy Bouchet, Rob Jack, Esteban Guevara, Takahiro Nemoto

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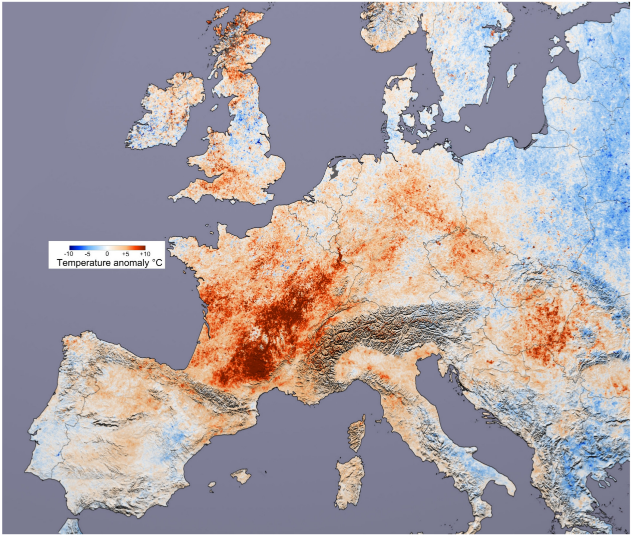
Why studying rare events?



2003 heat wave, Europe [Terra MODIS]

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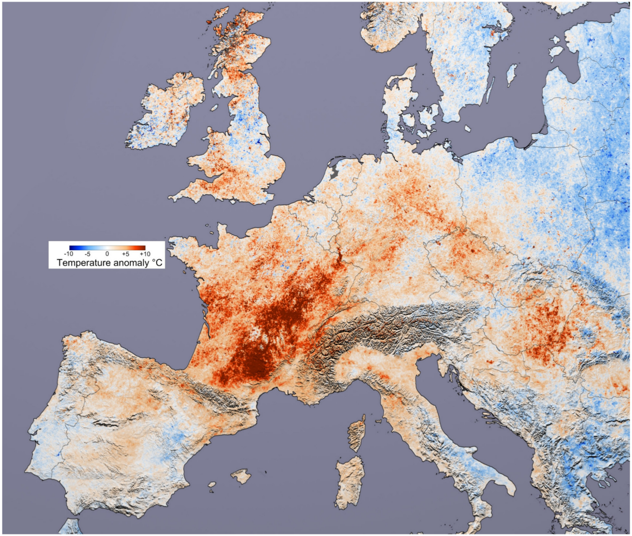

[Anomaly for **1-month** average] 2003 heat wave, Europe [Terra MODIS]

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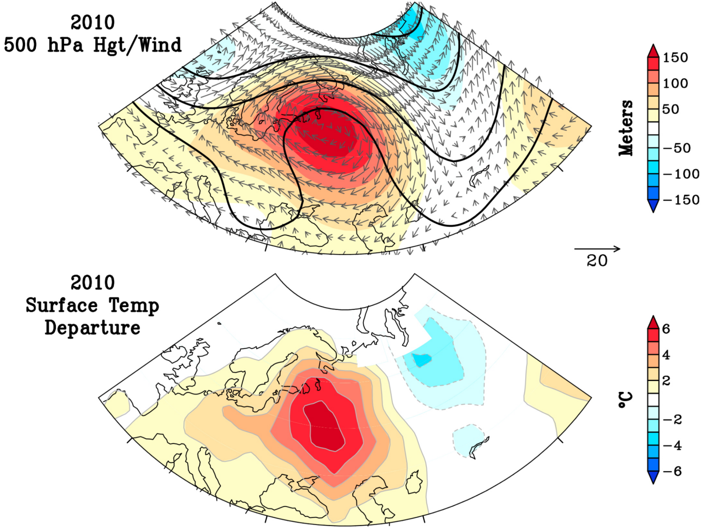

2010 heat wave in Western Russia [Dole *et al.*, 2011]

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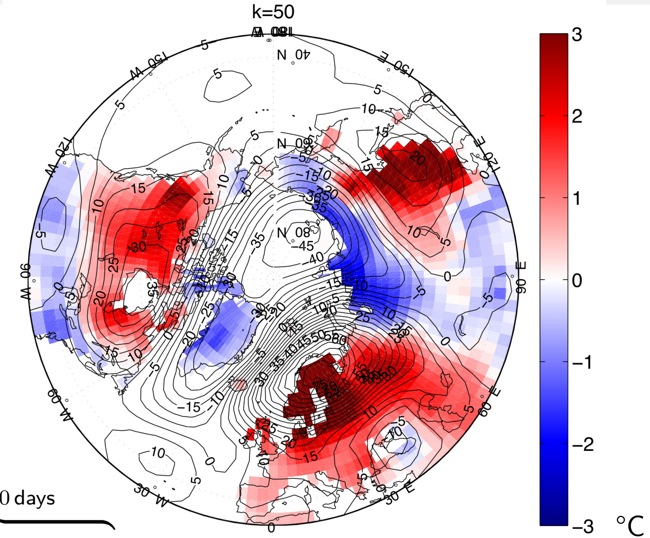

$$\overbrace{\frac{1}{t_{\max}}\int_0^{t_{\max}}}^{t_{\max}=40\ \text{days}} dt\,\Delta T(t) > 2^\circ C \Rightarrow$$ « Teleconnection patterns » [Bouchet *et al.*]

Why studying rare events?

Questions for physicists and mathematicians:

- Probability and **dynamics** of rare events?
- How to **sample** these in numerical modelisations?
- Numerical **tools and methods** to understand their formation?

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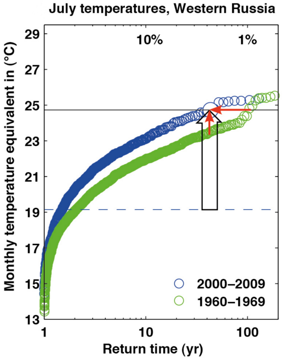


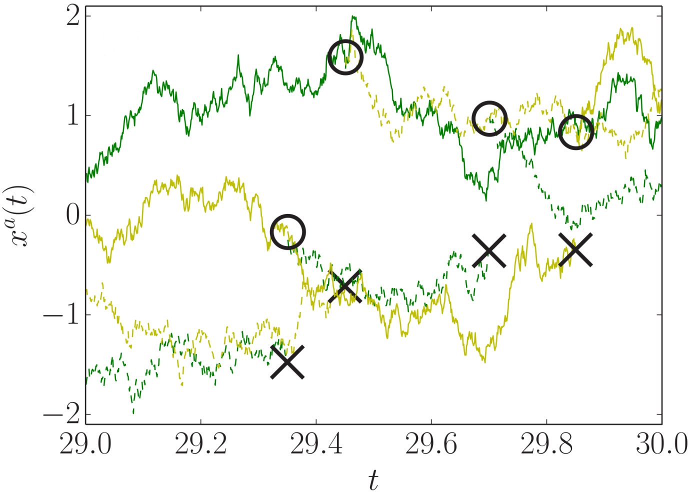
Evolution of the return time of the monthly averaged temperature

$$\frac{1}{t_{\max}} \int_0^{t_{\max}} dt \; T(t)$$

Due anthropogenic impact on climate?

[Otto *et al.*, 2012]

An example: 4 copies, 1 degree of freedom $\mathcal{C} = x \in \mathbb{R}$



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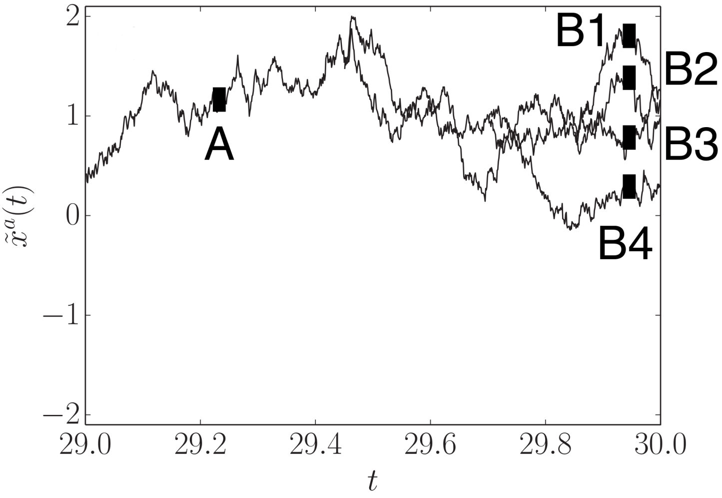
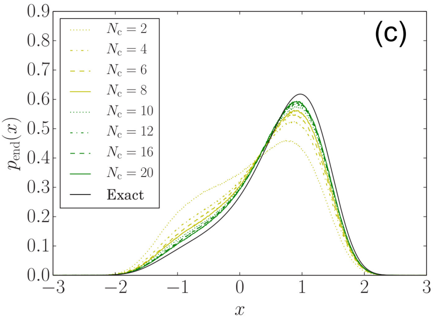
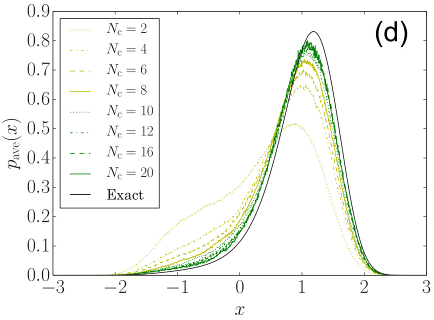


Huge sampling issue

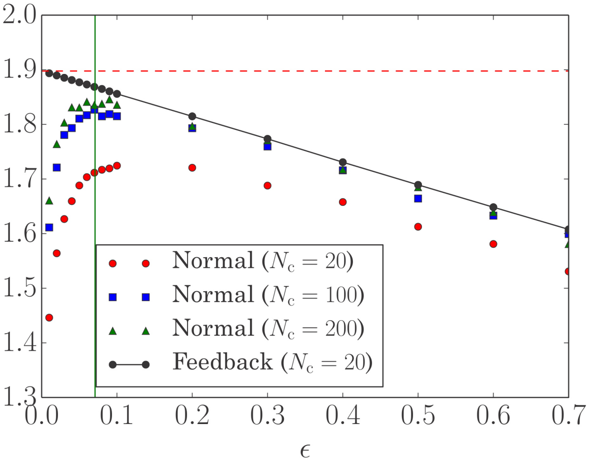
Example distributions for a simple Langevin dynamics





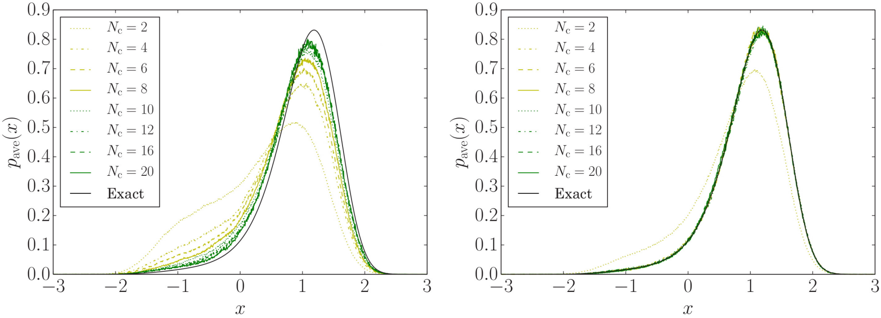
final-time: $p_{\mathrm{end}}(x)$ intermediate-time: $p_{\mathrm{ave}}(x)$

The small-noise crisis: systematic errors grow as $\epsilon \to 0$



Cause: as $\epsilon \to 0$, $p_{\text{ave}}(x)$ & $p_{\text{end}}(x) \to$ sharply peaked at *different points*
i.e. the clones do not ~~attack~~ sample correctly the phase space

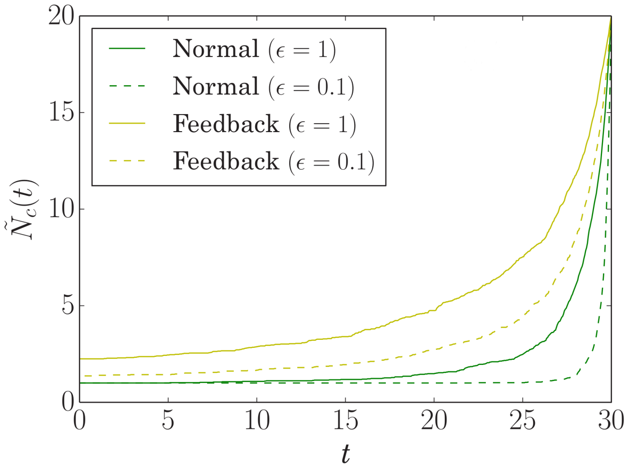
Improvement of the small-noise crisis



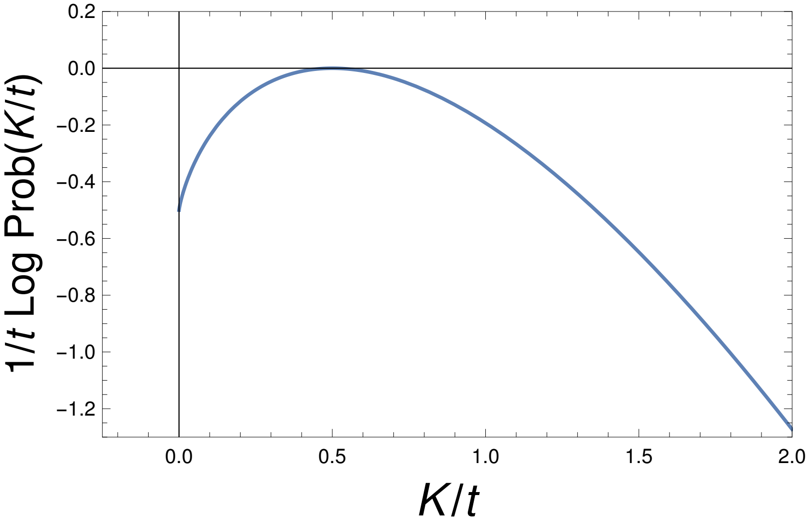
Much more efficient evaluation of the biased distribution.
Even for a very crude (polynomial) approximation of the effective force.

Questions: why is it working?

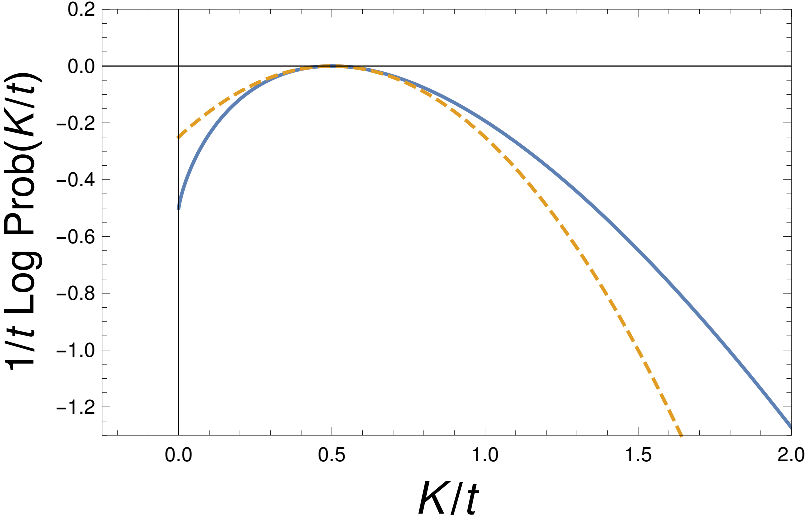
Improvement of the depletion-of-ancestors problem:



Dashed line: lower noise Continuous line: higher noise



$$\text{Prob}[K] \sim e^{t\,\varphi(K/t)}$$



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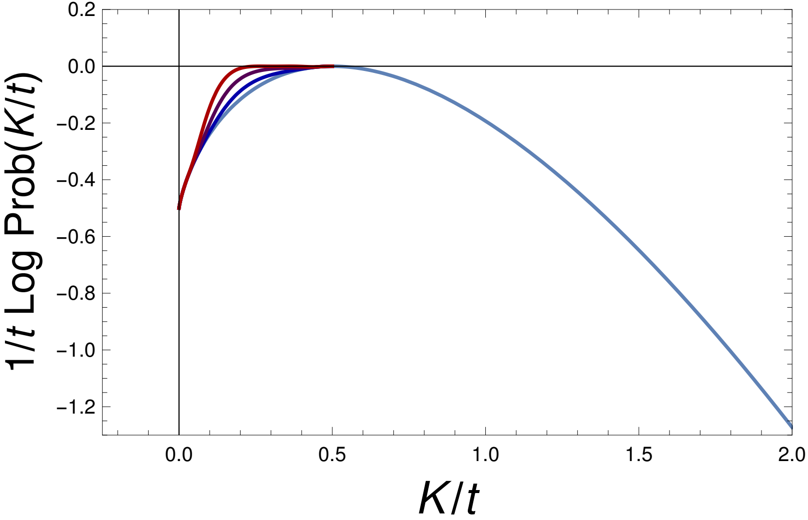

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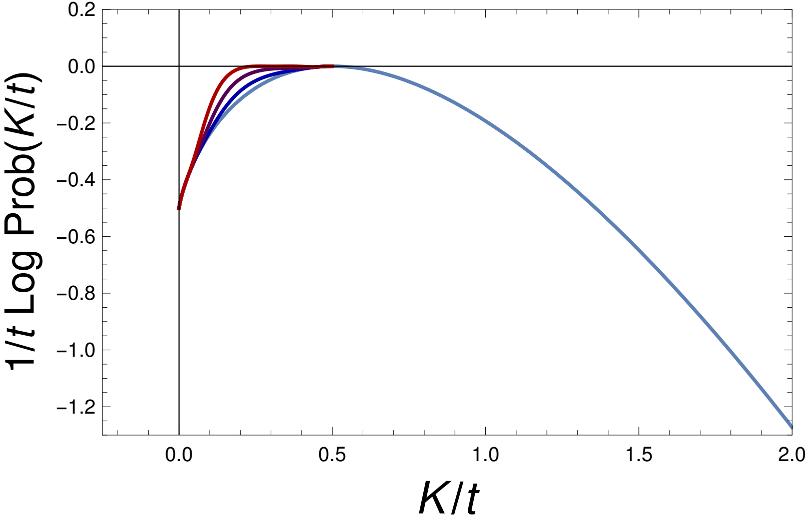
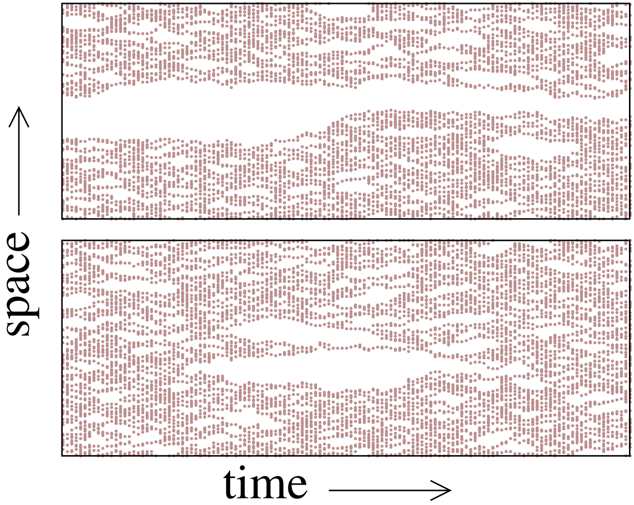
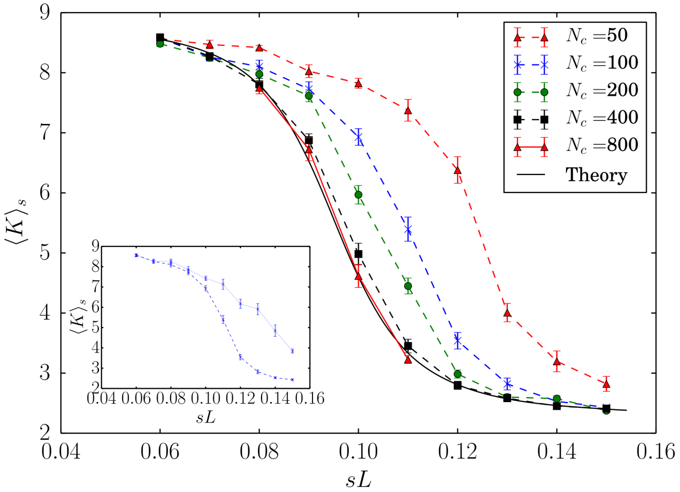


$\mathrm{Prob}[K] \sim e^{t\,\varphi(K/t)}$ Finite-time & -size scalings matter.



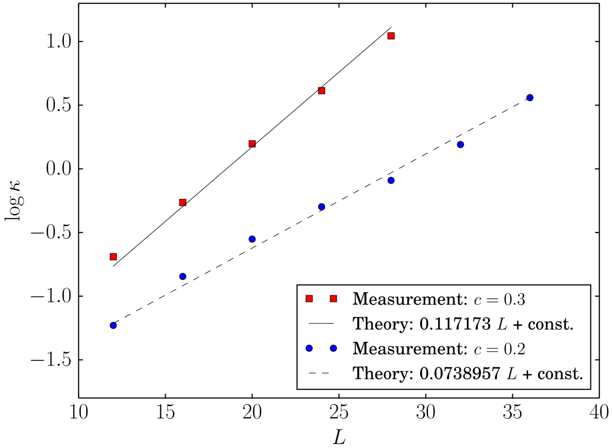
[Merrolle, Garrahan and Chandler, 2005]

Improvement of the small-noise crisis (ii)



Interacting system in 1D.

Effective force: 1-, 2-, 3- body interactions only [also crude approx.].



Exponential divergence of the susceptibility