Interfaces in Correlated Disorder: what we learn from the Directed Polymer

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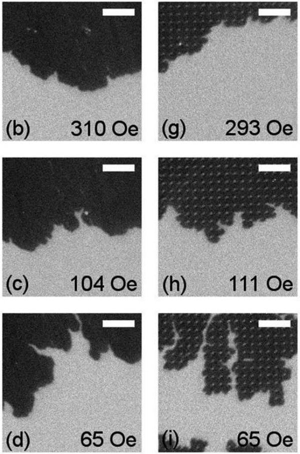
1D Interfaces

Interfaces in magnetic films



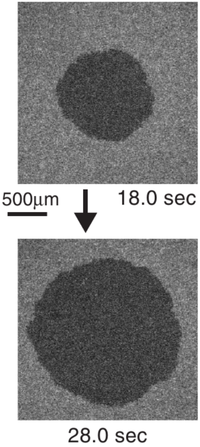
from Metaxas *et al.*

APL **94** 132504 (2009)

Large range of physical scales

Wide spectrum of phenomena

Growth in liquid crystals



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PRL **104** 230601 (2010)

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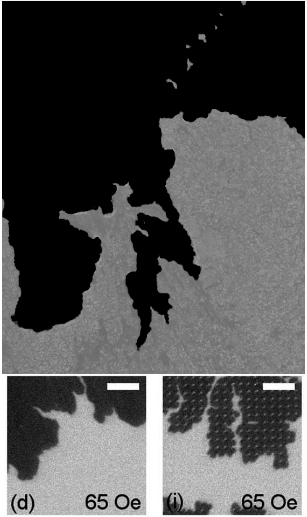

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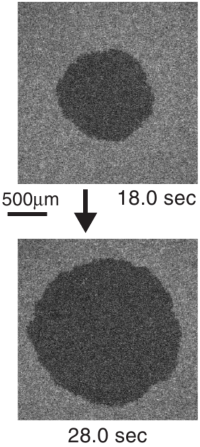

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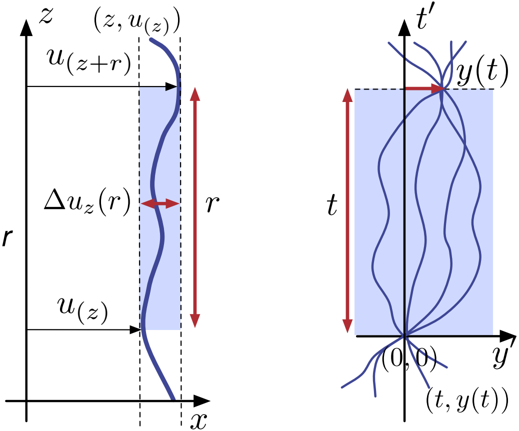
1D Interface in the Directed Polymer (DP) language

[Trick n° 1]

- No bubbles
- No overhangs
- Interface lengthscale *r*

$\updownarrow$

DP 'time' *t*



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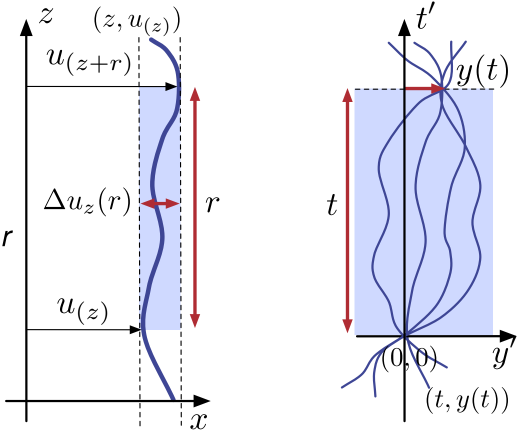

working at fixed 'time' *t* $\iff$ integration of fluctuations at scales smaller than *t*

$\boxed{\text{time duration} \equiv \text{lengthscale}}$

Disordered elastic systems

- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\mathrm{el}}[y(t'),t] = \frac{c}{2}\int_0^t dt' \, \left[\partial_{t'} y(t')\right]^2$$

- Disorder: tends to bend it

$$\mathcal{H}_V^{\mathrm{dis}}[y(t'),t] = \int_0^t dt' \; V\big(t', y(t')\big)$$

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$$\left.\right\} \mathcal{H}_V = \mathcal{H}^{\text{el}} + \mathcal{H}_V^{\text{dis}}$$

$$\boxed{\text{Competition btw “order” and “disorder”}}$$

- Ingredients up to now:

elastic constant c disorder potential $V(t,y)$ $\overbrace{\text{temperature } T}^{\text{trajectory weight } \propto e^{-\mathcal{H}_V/T}}$

Questions

- Nature of fluctuations
  - ⋆ $V(t, y) \equiv 0$: *diffusive* ($y \sim t^{1/2}$), **Edwards-Wilkinson** (EW)
  - ⋆ $V(t, y) \not\equiv 0$: *super-diffusive* ($y \sim t^{2/3}$), **Kardar-Parisi-Zhang** (KPZ)

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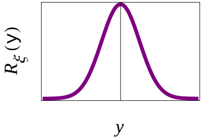
- Role of (experimentally ineluctable) **disorder correlations**?

  zero mean, Gaussian, $\overline{V(t,y)V(t',y')} = D\delta(t'-t)R_\xi(y'-y)$



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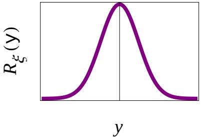

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- Summary of ingredients:

  elastic constant c temperature T disorder $\Big|$ amplitude D, corr. length ξ

Free-energy fluctuations [Tricks n°2&3]

- Partition function Z_V *vs.* Free-energy F_V

$$Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t') e^{-\frac{1}{T}\mathcal{H}_V[y(t'),t]} \qquad F_V(t,y) = -\frac{1}{T}\log Z_V(t,y)$$

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- **Statistical Tilt Symmetry**

$$\boxed{F_V(t,y) = \underbrace{c\frac{y^2}{2t} + \frac{T}{2}\log\frac{2\pi Tt}{c}}_{\substack{\text{thermal contribution}\\ F_{V\equiv 0}}} + \underbrace{\bar{F}_V(t,y)}_{\substack{\text{disorder}\\ \text{contribution}}}} \qquad \textbf{(STS)}$$

- **Tilted** KPZ equation for $\bar{F}_V(t,y)$

$$\partial_t \bar{F}_V + \frac{y}{t}\partial_y \bar{F}_V = \frac{T}{2c}\partial_y^2 \bar{F}_V - \frac{1}{2c}\left[\partial_y \bar{F}_V\right]^2 + V(t,y)$$

Non-linear, additive noise, $\bar{F}_V(0,y) \equiv 0$: “simple” initial cond.

Known results @$\xi = 0$ $\quad [\Longleftrightarrow T \to \infty$ @$\xi > 0]$

- **Central tool**: 2-point correlation function

$$\bar{R}(t, y_2 - y_1) = \overline{\partial_y \bar{F}_V(t, y_1) \partial_y \bar{F}_V(t, y_2)}$$

- **Infinite-'time' limit** (steady state)

$\bar{F}(t = \infty, y)$ distributed as a Brownian Motion

i.e. : $Prob\big[\bar{F}(t = \infty, y)\big]$ Gaussian, of correlator

$$\bar{R}(t = \infty, y) = \widetilde{D}_{\xi=0}\, \delta(y) \quad \text{with} \quad \boxed{\widetilde{D}_{\xi=0} = \frac{cD}{T}}$$

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- **Roughness** function $B(t)$ [variance of end-point fluct.]

$$B(t) = \overline{\langle y(t)^2 \rangle} = \overline{\frac{\int dy\, y^2 Z_V(t,y)}{\int dy\, Z_V(t,y)}}$$

$$\boxed{B(t) = \big[\widetilde{D}_{\xi=0}\, / c^2\big]^{2/3} t^{4/3}} \quad \text{as } t \to \infty$$

Effective model @$\xi > 0$ & numerical results

$\xi > 0$ not obtained from perturbation of $\xi = 0$

- **Distribution** of free-energy

scales closely to the $\xi = 0$ case

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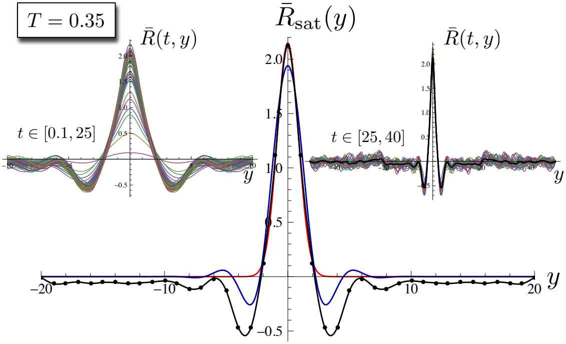
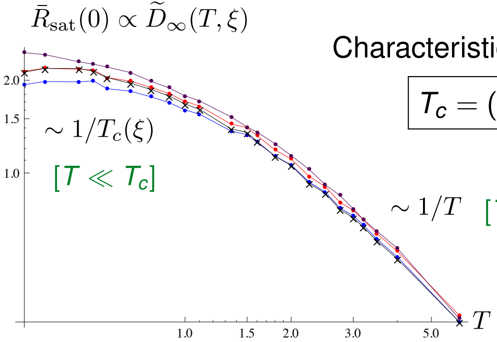
- **2-point** correlation function of amplitude $\widetilde{D}$

$$\boxed{\bar{R}(t, y) \simeq \widetilde{D} R_\xi(y) \text{ as } t \to \infty}$$



High- and low-temperature regimes



Characteristic temperature

$$\boxed{T_c = (\xi c D)^{1/3}}$$

- (Advanced) **scaling** analysis

$T \ll T_c$	$T \gg T_c$
one optimal trajectory	many trajectories
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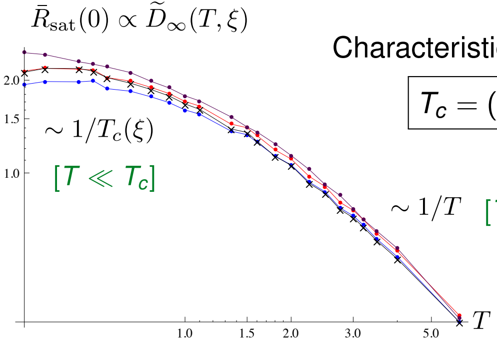

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Summary & open questions [arXiv:1209.0567]

- **Geometry** of interface $\longleftrightarrow$ Directed Polym. **free-energy** fluctuat.
  - $\star$ $T \lesssim T_c$: ξ **plays a role at all lengthscales** $[T_c = (\xi c D)^{1/3}]$
  - $\star$ focus on the free-energy 2-point correlator amplitude $\widetilde{D}$
  - $\star$ understanding of 'time'- (i.e. length) multiscaling

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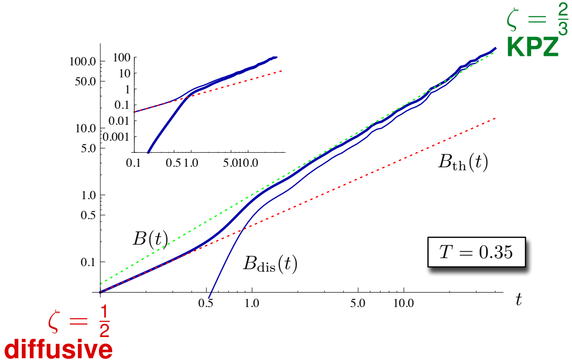

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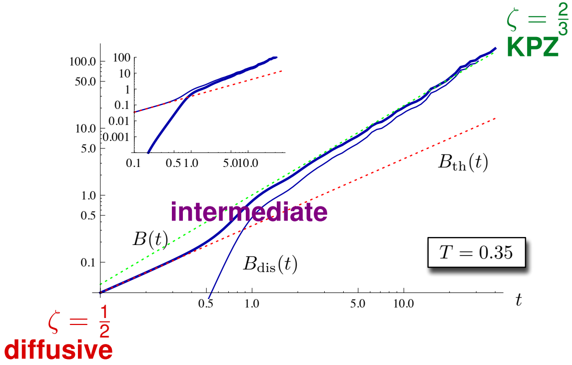

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- Perspective
  - ⋆ experimental probe of the importance of ξ
  - ⋆ interpretation in other 'incarnations' of the KPZ class
    - . growth interfaces with $F(t, y) =$ height at (real) time t
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  - ⋆ **creep law**: non-linear response to small force

$$\text{velocity} \sim \exp\left\{ - \left[\frac{\overbrace{\text{critical force}}^{\text{depends on } c,\, D,\, T,\, \xi}}{\text{force}}\right]^{1/4} \right\}$$

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$$\partial_t Z_V = \Big[\frac{T}{2c}\partial_y^2 - \frac{1}{T}V(t,y)\Big] Z_V(t,y) \qquad \textbf{(SHE)}$$

Linear, multiplicative noise, reversible

- **Kardar-Parisi-Zhang** equation

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