Depinning transition for domain walls with an internal degree of freedom

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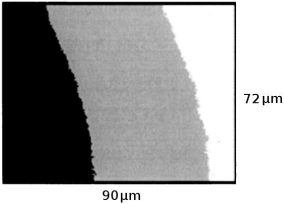
Outline

1. Interface Physics
   - Systems
   - Depinning transition
   - Experiments
2. Depinning with internal degree of freedom
   - Modelisation
   - Dynamics

Magnetic domain wall



from Lemerle *et al.*, PRL **80** 849 (1998)

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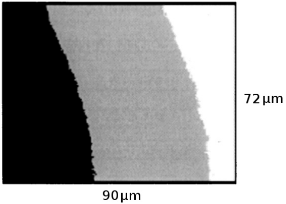

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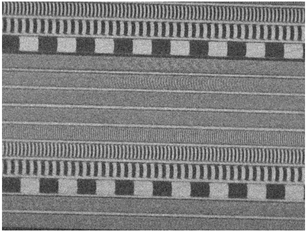
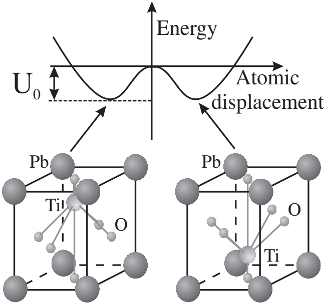
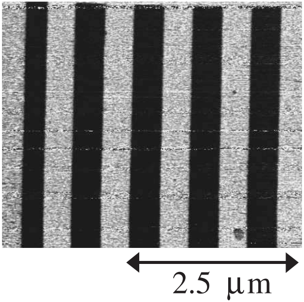


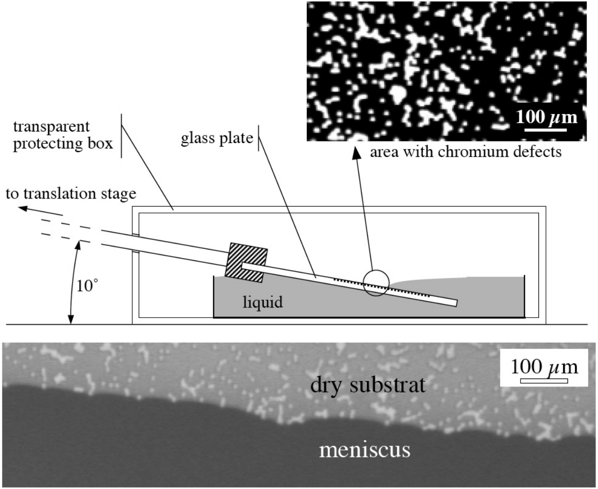
Ferroelectric domain wall





from Paruch *et al. J. Appl. Phys*, **100** 051608 (2006)

Contact line of a fluid



from Moulinet, Guthmann and Rolley, *Eur. Phys. J. E*, **8** 437 (2002)

Common underlying description?

Large range of physical scales

- magnetic/ferroelecric domain walls
- contact line
- growth interfaces
- crack propagation

Questions

- Statics

  fluctuations, roughness
- Non-equilibrium dynamics

  motion of the interface
- Nature, role of disorder

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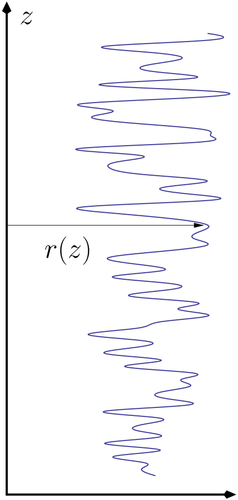
Disordered elastic systems

- Elasticity: tends to flatten the interface

$$\frac{c}{2}\int dz\,\left(\nabla r(z)\right)^2$$

- Disorder: tends to bend it

$$\int dz\,V\left(r(z),z\right)$$



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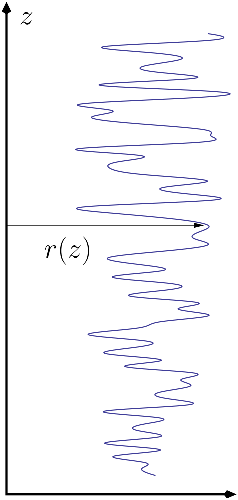

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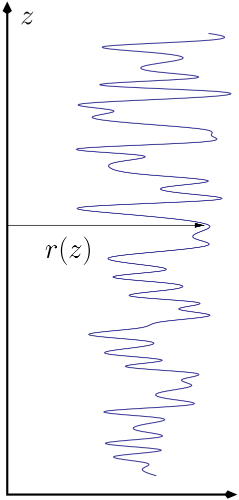
Competition btw “order” and “disorder”



Is $r(z)$ enough?

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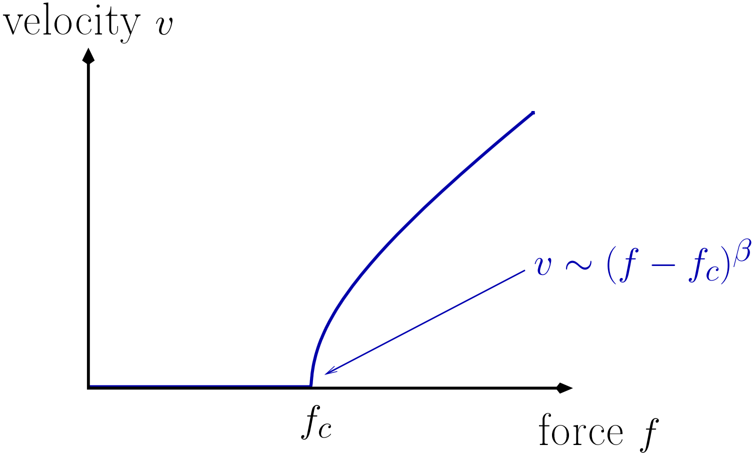
$\longrightarrow$ Have a look to the dynamics in simple examples.

Depinning transition @ zero temperature



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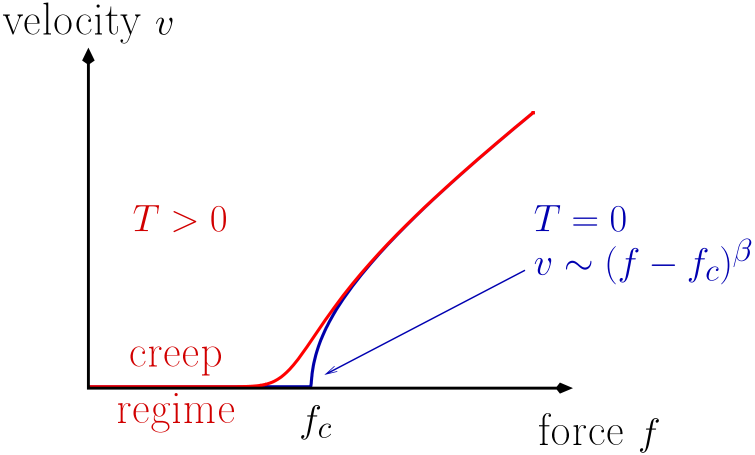
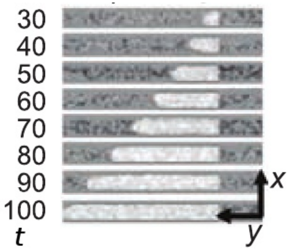


Comparison with experiment: ferromagnetic wire

$$v(f) \sim \exp\left[-\frac{U_c}{T}\left(\frac{f_c}{f}\right)^{\mu}\right] \qquad \text{(creep)}$$

	Field drive		Current drive	
	μ*	σ*	μ	σ
Experiment	1.2 ± 0.1	1.4 ± 0.1	0.33 ± 0.06	2.0 ± 0.2
Theory	1.0	1.5	0.5	1.25



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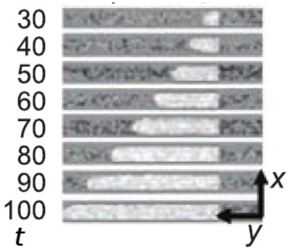

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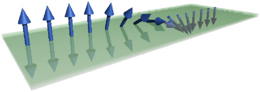
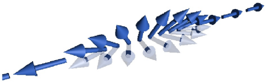
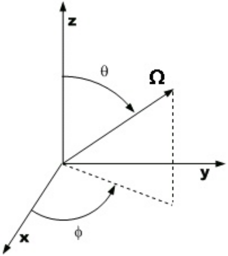
Bulk model







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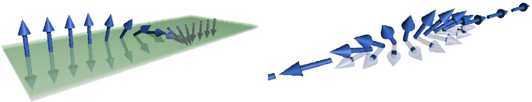
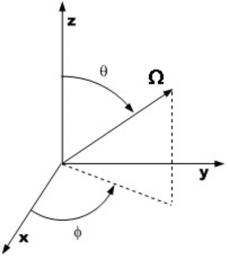




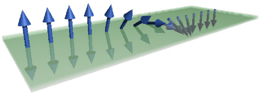
- Bulk energy

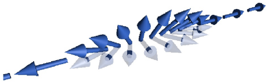
$$E = \int d^dx \left\{ J\left[(\nabla\theta)^2 + \sin^2\theta(\nabla\phi)^2\right] + K \sin^2\theta + K_\perp \sin^2\theta \cos^2\phi \right\}$$

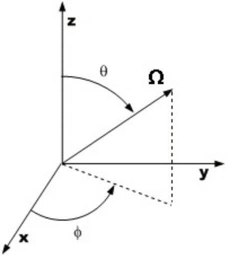
- Equation of motion

$$\big(\partial_t + v_s \cdot \nabla\big)\Omega = \Omega \times \big(\frac{\delta E}{\delta \Omega} + f + \eta\big) - \Omega \times \big(\alpha\partial_t + \beta v_s \cdot \nabla\big)\Omega$$

Bulk model







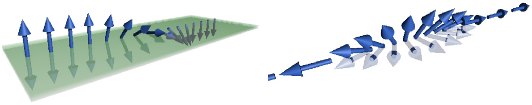
- Rigid wall approximation

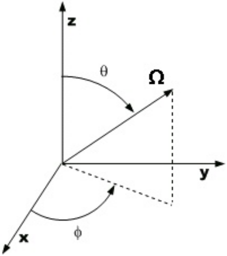
$$\alpha\partial_t r - \partial_t\phi = f + \text{Landscape} \qquad + \eta_1$$
$$\alpha\partial_t\phi + \partial_t r = -\frac{1}{2}K_\perp \sin 2\phi \qquad + \eta_2$$

- Effective model:

Position $r(t)$ coupled to phase $\phi(t)$.

Bulk model





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- Effective model:

  Position $r(t)$ coupled to phase $\phi(t)$.

- One chooses: Landscape $= f - \cos\kappa r$

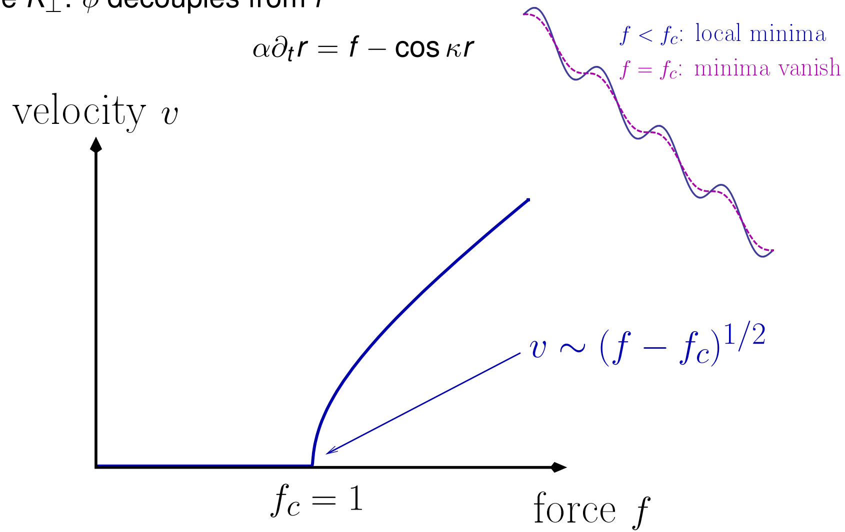
Depinning @

Large $K_\perp$: ϕ decouples from r

Depinning @ zero temperature

Large $K_\perp$: ϕ decouples from r

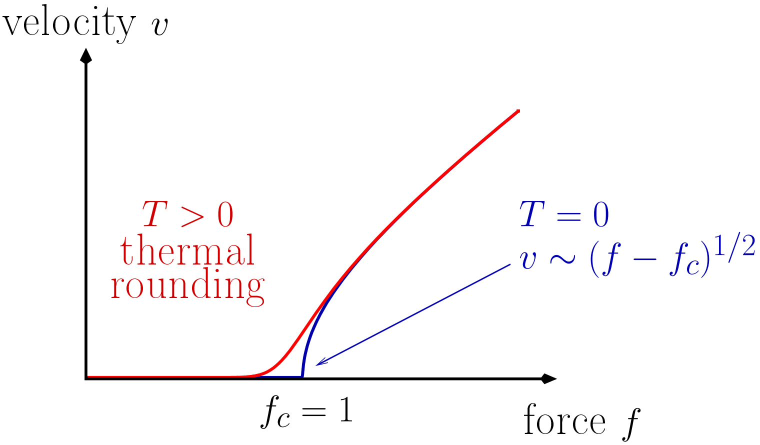
$$\alpha \partial_t r = f - \cos \kappa r$$



Depinning @ finite temperature

Large $K_\perp$: ϕ decouples from r

$$\alpha \partial_t r = f - \cos \kappa r + \eta$$



Depinning @ zero temperature

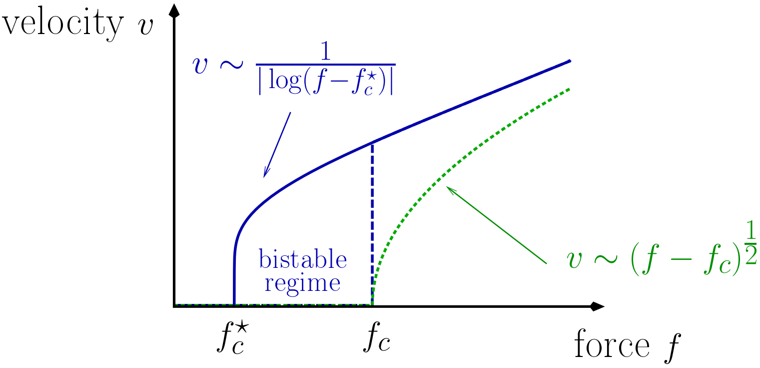
Smaller $K_\perp$: ϕ matters

$$\alpha\partial_t r - \partial_t\phi = f - \cos\kappa r$$
$$\alpha\partial_t\phi + \partial_t r = -\frac{1}{2}K_\perp \sin 2\phi$$

Depinning @ zero temperature

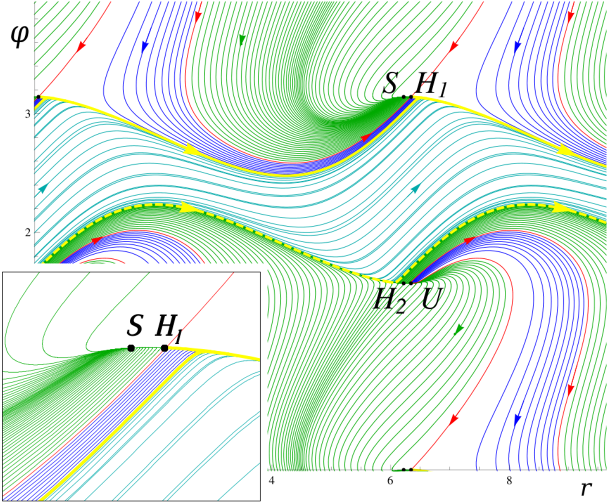
Smaller $K_\perp$: ϕ matters

- Dramatic change in the depinning law: $v \sim \frac{1}{|\log(f - f_c^\star)|}$



- Depinning at lower critical force: $f_c^\star < f_c$
- Bistability

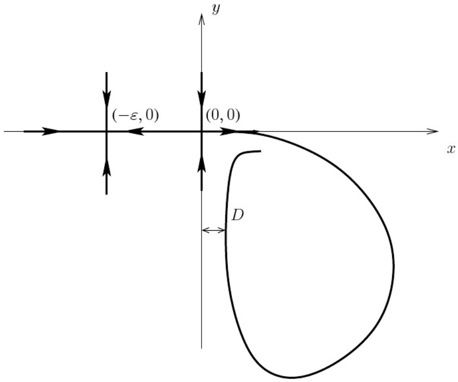
Phase space



In the bistable regime ($f_c^\star < f < f_c$)

Phase space

Homoclinic bifurcation:



$$f > f_c^\star$$

Phase space

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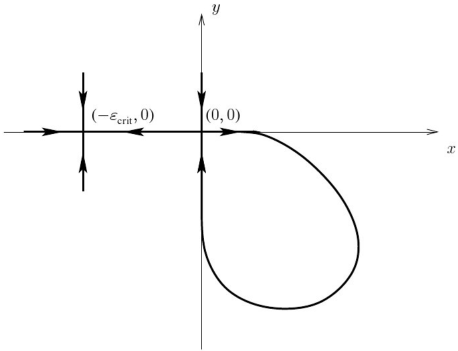
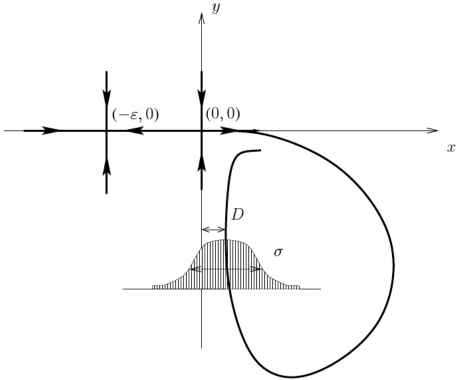


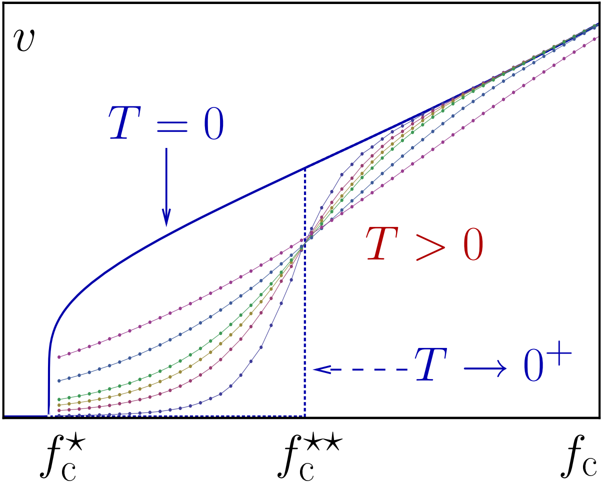
$$f = f_c^\star$$

Finite temperature



$$\text{escape time} \sim \underbrace{\exp\left(\frac{\epsilon^3}{T}\right)}_{\text{Arrhenius}} \underbrace{\exp\left(-\frac{A}{T}(\epsilon-\epsilon_c)^2\right)}_{\text{Trapping probability}}$$

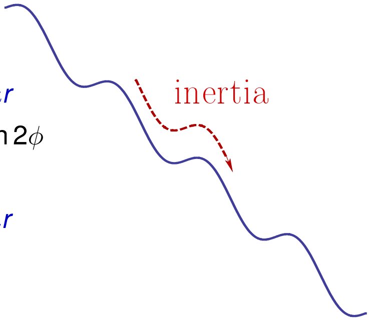
Finite temperature



Force-velocity characteristics

Analogy

$$\alpha\partial_t r - \partial_t\phi = f - \cos\kappa r$$
$$\alpha\partial_t\phi + \partial_t r = -\tfrac{1}{2}K_\perp \sin 2\phi$$
$$\updownarrow$$
$$m\partial_t^2 r + \alpha\partial_t r = f - \cos\kappa r$$



The phase ϕ plays the role of a velocity:

inertia helps to cross barriers

[see also Risken chap.11]

Analogy

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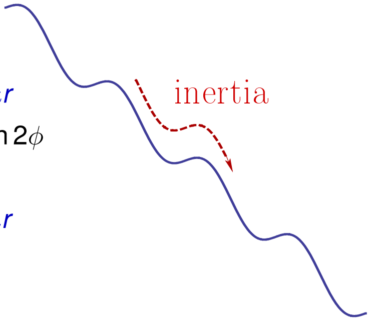

BUT ...

Analogy

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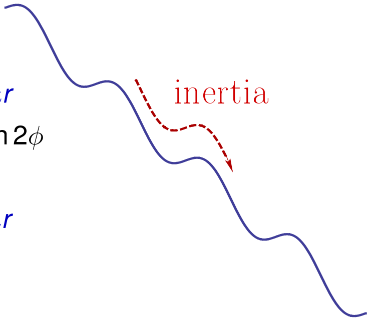
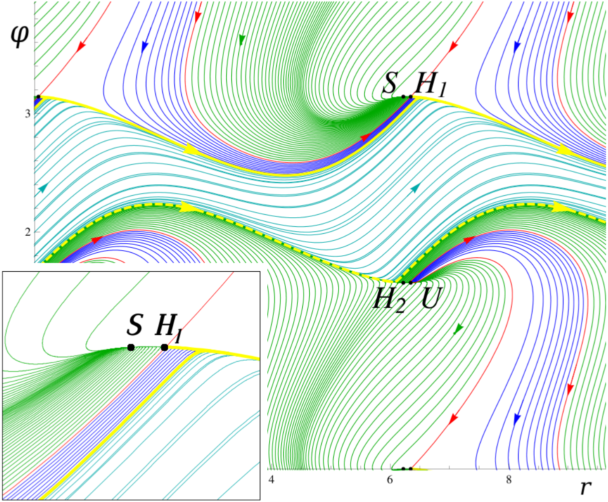


the velocity is unbounded WHEREAS ϕ is bounded and periodic

Topological transition



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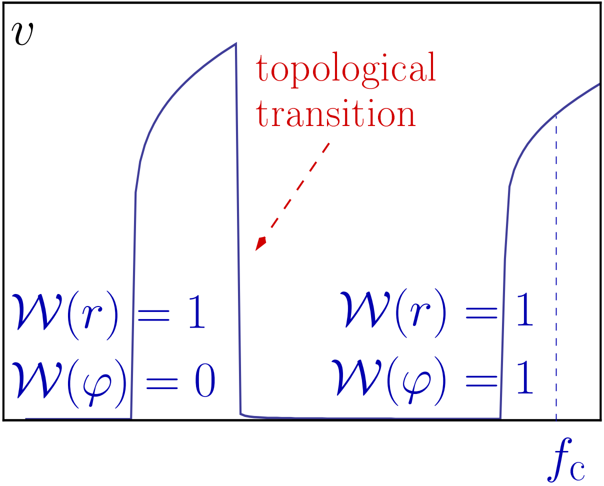
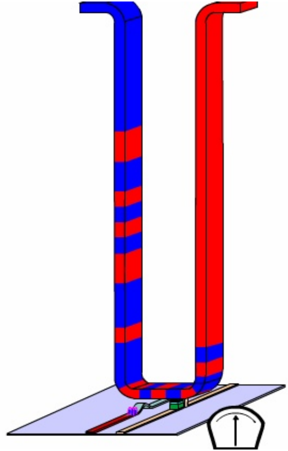
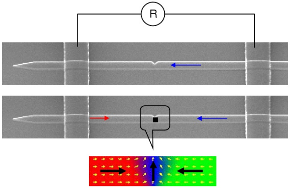


Successive regimes characterized by winding numbers $\mathcal{W}$

Experiment



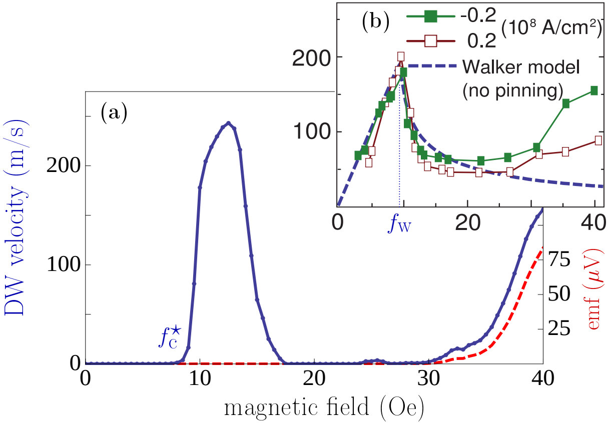
SPINTRONICS



from Parkin *et al.*, Science **320** 190 (2008)

Experiment

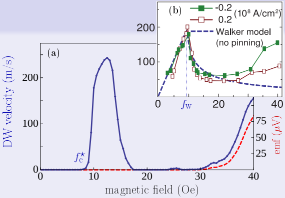
experiment from Parkin *et al.*, Science **320** 190 (2008)



Outlook

Internal degree of freedom

- unusual depinning law
- bistability
- non-monotonous $v(f)$ at finite T
- link with experiments



Perspective

- Current driven wall
- Interface with elasticity

→ modified creep law?

- Experiments

periodic patterning

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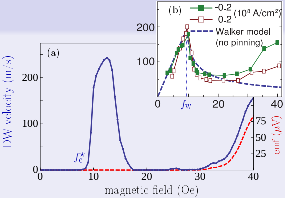

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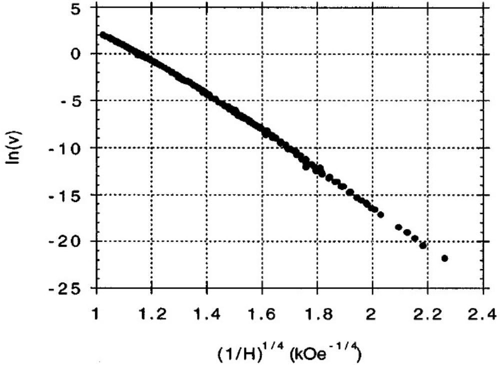

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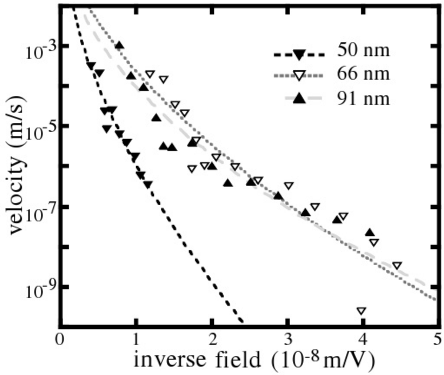
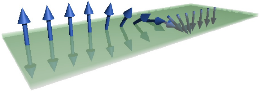
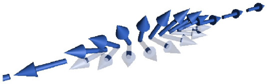
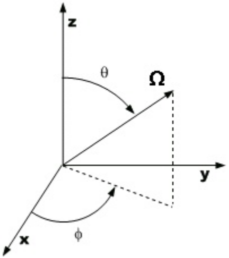


from Paruch *et al.*, PRL **94** 197601 (2005)

Bulk model







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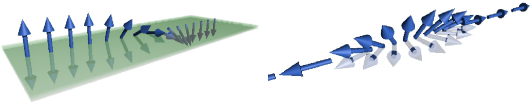

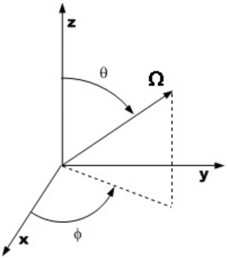

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- Equation of motion

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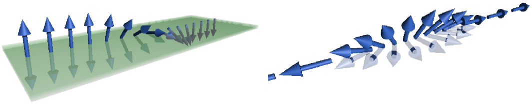
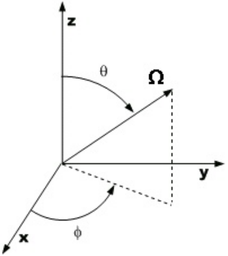


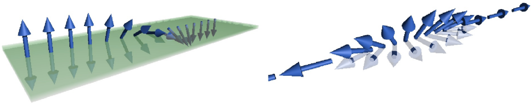


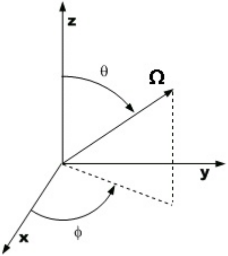
- Expanded equations of motion

$$\partial_t\theta - \alpha \sin\theta\, \partial_t\phi + v_s\big(\partial_x\theta - \beta \sin\theta\, \partial_x\phi\big) = -\frac{1}{2}K_\perp \sin\theta \sin 2\phi - \frac{J}{\sin\theta}\partial_x\big(\sin^2\theta\, \partial_x\,\phi\big)$$

$$\sin\theta\, \partial_t\phi + \alpha\partial_t\theta + v_s\big(\sin\theta\, \partial_x\phi + \beta\partial_x\theta\big) = -\frac{1}{2}K_\perp \sin 2\theta\, \cos^2\phi - \frac{1}{2}\big(K + J(\partial_x\phi)^2\big)\, \sin 2\theta - H_{\text{ext}} \sin\theta + J\partial_x^2\theta$$

Bulk model





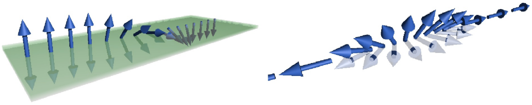
- Solitonic solution [Walker 1974]

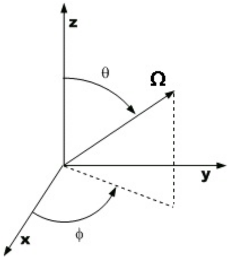
$$\sin 2\phi(x,t) = \frac{f}{f_W} \qquad f_W = \frac{1}{2}\alpha K_\perp$$

$$\theta(x,t) = 2 \arctan \exp \left\{ \left[1 + \frac{K_\perp}{K}\cos^2\phi\right]^{\frac{1}{2}} \left(\sqrt{\frac{K}{J}}\, x - vt \right) \right\}$$

$$v = \frac{H_{\text{ext}}}{H_c}\left[1 + \frac{K_\perp}{K}\cos^2\phi\right]^{-\frac{1}{2}}$$

Bulk model





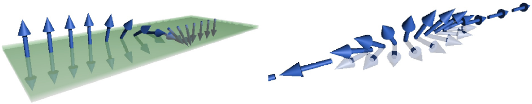
- Rigid wall approximation

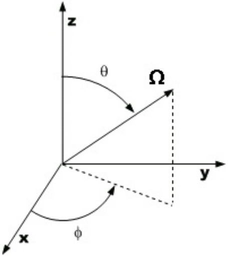
$$\alpha\partial_t r - \partial_t \phi = f + \text{Landscape} \qquad + \eta_1$$

$$\alpha\partial_t \phi + \partial_t r = -\frac{1}{2}K_\perp \sin 2\phi \qquad + \eta_2$$

Unit of length: $\sqrt{J/K}$

Bulk model





- Rigid wall approximation

$$\alpha\partial_t r - \partial_t \phi = f - \cos \kappa r \qquad + \eta_1$$

$$\alpha\partial_t \phi + \partial_t r = -\frac{1}{2} K_\perp \sin 2\phi \qquad + \eta_2$$

- Effective model:

  Position $r(t)$ coupled to phase $\phi(t)$.