Interfaces in random media: scaling in and out of equilibrium.

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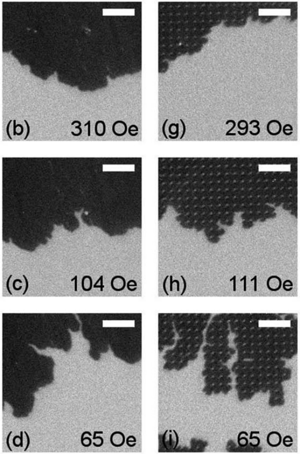
1D Interfaces

Interfaces in magnetic films



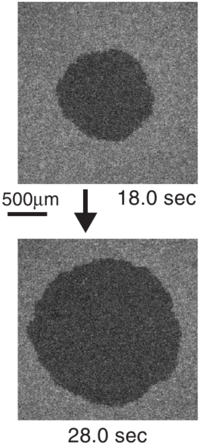
from Metaxas *et al.*

APL **94** 132504 (2009)

Large range of physical scales

Wide spectrum of phenomena

Growth in liquid crystals



from Takeuchi & Sano

PRL **104** 230601 (2010)

Disordered elastic systems

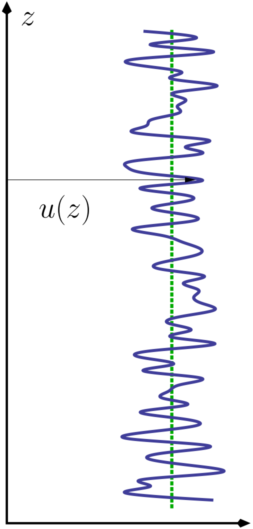
- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\text{el}} = \frac{c}{2}\int dz\,\big(\nabla u(z)\big)^2 \qquad \text{[Short-range]}$$

$$\mathcal{H}^{\text{el}} = \frac{c}{2\pi}\int dz dz' \,\frac{\big(u(z)-u(z')\big)^2}{(z-z')^2} \qquad \text{[Long-range]}$$

- Disorder: tends to bend it

$$\mathcal{H}^{\text{dis}}_V = \int dz\, V\big(u(z),z\big)$$



Competition btw "order" and "disorder"

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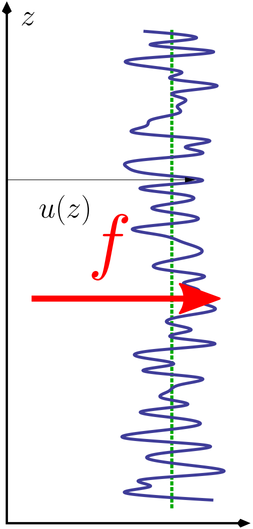
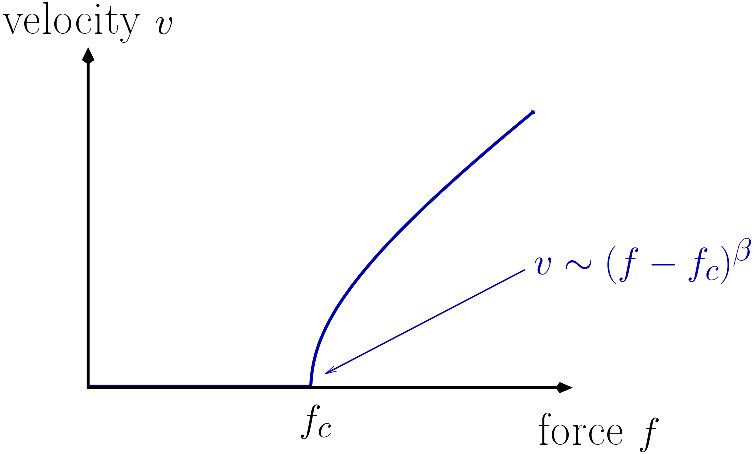
- Force: induces motion of the interface



Competition btw “order” and “disorder”

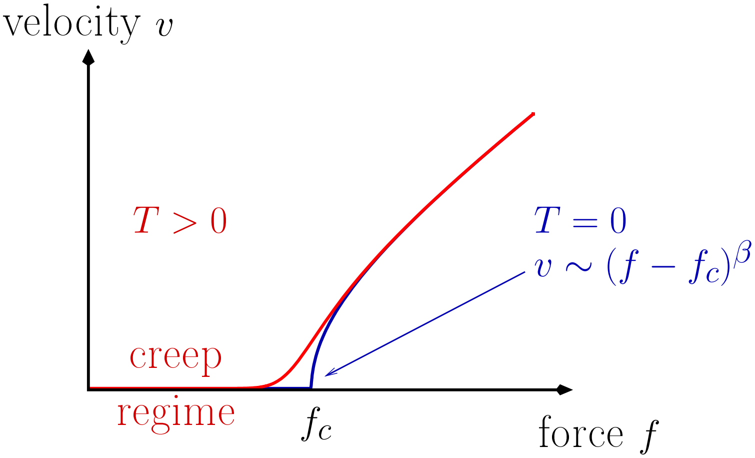
Depinning transition @ zero temperature

threshold force f_c



Depinning transition @ finite temperature

thermal rounding
creep regime

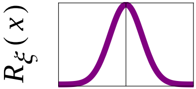


Uncorrelated disorder:

$$\overline{V(z,x)\,V(z',x')} = D\,\delta(z'-z)\delta(x'-x)$$

Correlated disorder on a lengthscale ξ:

$$\overline{V(z,x)\,V(z',x')} = D\,\delta(z'-z)R_\xi(x'-x)$$



scaling as $R_\xi(x) = \frac{1}{\xi}\,R_{\xi=1}(x/\xi)$

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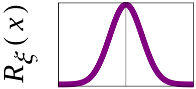
Can ξ play a role at lengthscales $\gg \xi$?



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Study 1D models with correlated disorder $(\xi > 0)$

1. Static properties
   short-range elasticity
   $\longrightarrow$ **Identification of lengthscales and power laws**
   [Elisabeth Agoritsas, Thierry Giamarchi, VL]

Study 1D models with correlated disorder $(\xi > 0)$

1. Static properties ($T > 0$ and $T \to 0$)
   short-range elasticity
   $\longrightarrow$ **Identification of lengthscales and power laws**
   [Elisabeth Agoritsas, Thierry Giamarchi, VL]

2. Dynamical properties ($T \to 0$)
   effective description
   $\longrightarrow$ **Creep regime ; crossover with linear response**
   [Reinaldo García-García, Elisabeth Agoritsas, Lev Truskinovsky, Damien Vandembroucq, VL]

Focus on scaling analysis, **beyond naive power counting**.

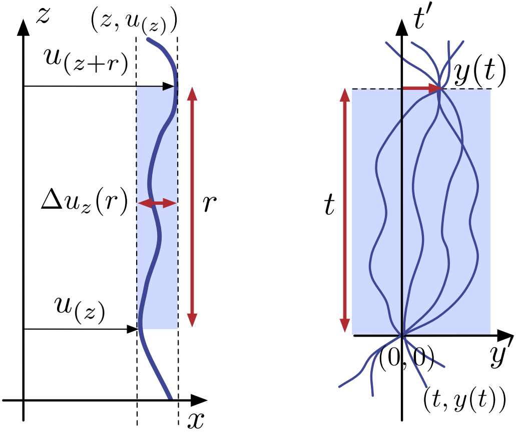
1D Interface in the Directed Polymer (DP) language

[Step n°1]

- No bubbles
- No overhangs
- Interface lengthscale r

$\updownarrow$

DP 'time' t



working at fixed 'time' t $\iff$
integration of fluctuations at scales smaller than t

lengthscale $\equiv$time duration

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Competition btw "order" and "disorder"

- Ingredients up to now:

elastic constant c disorder potential $V(t,y)$ $\overbrace{\text{temperature } T}^{\text{trajectory weight } \propto e^{-\mathcal{H}_V/T}}$

Free-energy fluctuations [Step n°2&3]

- Partition function Z_V *vs.* Free-energy F_V

$$Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t') e^{-\frac{1}{T}\mathcal{H}_V[y(\cdot),t]} \qquad F_V(t,y) = -\frac{1}{T}\log Z_V(t,y)$$

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- **Statistical Tilt Symmetry**

$$F_V(t,y) = \underbrace{c\frac{y^2}{2t} + \frac{T}{2}\log\frac{2\pi Tt}{c}}_{\substack{\text{thermal contribution} \\ F_{V\equiv 0}}} + \underbrace{\bar{F}_V(t,y)}_{\text{disorder contribution}} \qquad \textbf{(STS)}$$

- **Tilted** KPZ equation for $\bar{F}_V(t,y)$

$$\partial_t \bar{F}_V + \frac{y}{t}\partial_y \bar{F}_V = \frac{T}{2c}\partial_y^2 \bar{F}_V - \frac{1}{2c}\left[\partial_y \bar{F}_V\right]^2 + V(t,y)$$

Non-linear, additive noise, $\bar{F}_V(0,y) \equiv 0$: "simple" initial cond.

Known results $@\xi = 0$ $[\iff T \to \infty \; @\xi > 0]$

- **Central tool**: 2-point correlation function

$$\bar{R}(t, y_2 - y_1) = \overline{\partial_y \bar{F}_V(t, y_1) \partial_y \bar{F}_V(t, y_2)}$$

- **Infinite-'time' limit** (steady state)

$\bar{F}(t = \infty, y)$ distributed as a Brownian Motion

i.e.: $Prob\big[\bar{F}(t = \infty, y)\big]$ Gaussian, of correlator

$$\bar{R}(t = \infty, y) = \widetilde{D}_{\xi=0}\, \delta(y) \quad \text{with} \quad \boxed{\widetilde{D}_{\xi=0} = \frac{cD}{T}}$$

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- **Roughness** function $B(t)$ [variance of end-point fluct.]

$$B(t) = \overline{\langle y(t)^2 \rangle} = \overline{\frac{\int dy\, y^2 Z_V(t,y)}{\int dy\, Z_V(t,y)}}$$

$$\boxed{B(t) = \big[\widetilde{D}_{\xi=0} / c^2\big]^{2/3} t^{4/3}} \quad \text{as } t \to \infty$$

Effective model @$\xi > 0$ & numerical results

$\xi > 0$ not obtained from perturbation of $\xi = 0$

- **Distribution** of free-energy

  scales closely to the $\xi = 0$ case

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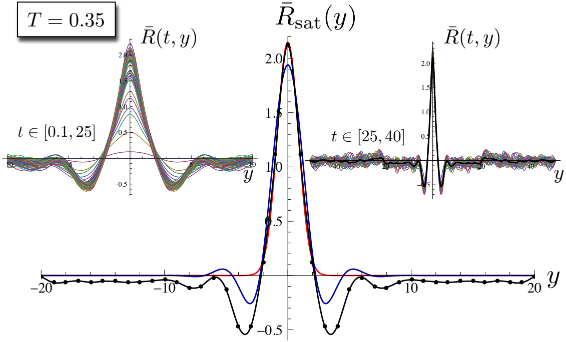
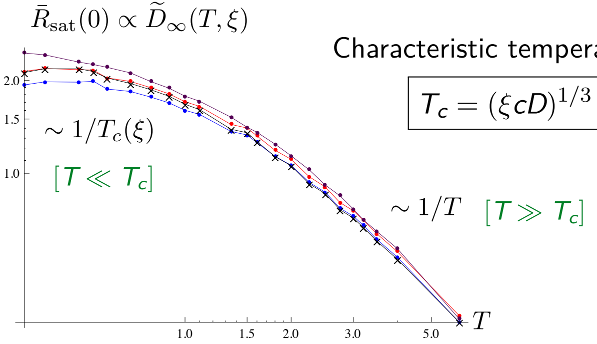
- **Distribution** of free-energy
  scales closely to the $\xi = 0$ case
- **2-point** correlation function of amplitude $\widetilde{D}$

$$\bar{R}(t, y) \simeq \widetilde{D} R_\xi(y) \text{ as } t \to \infty$$



High- and low-temperature regimes



Characteristic temperature

$$T_c = (\xi c D)^{1/3}$$

- (Advanced) **scaling** analysis

$T \ll T_c$	$T \gg T_c$
one optimal trajectory	many trajectories
$\widetilde{D} = \dfrac{cD}{T_c}$	$\widetilde{D} = \dfrac{cD}{T}$

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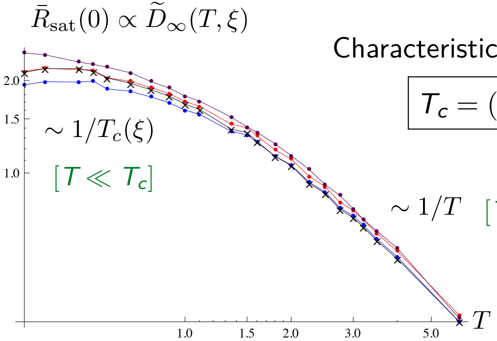

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- (Advanced) **scaling** analysis [Note: again $B(t) = \overbrace{\left[\widetilde{D}/c^2\right]^{2/3}}^{\xi \text{ plays a role at all scales}} t^{4/3}$]

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Lengthscales & dynamics

PRE 87 042406 (2013)

- **Geometry** of interface $\longleftrightarrow$ Directed Polym. **free-energy** fluctuat.
  - $T \lesssim T_c$: ξ **plays a role at all lengthscales** $[T_c = (\xi c D)^{1/3}]$
  - focus on the free-energy 2-point correlator amplitude $\widetilde{D}$
  - understanding of 'time'- (i.e. length) multiscaling

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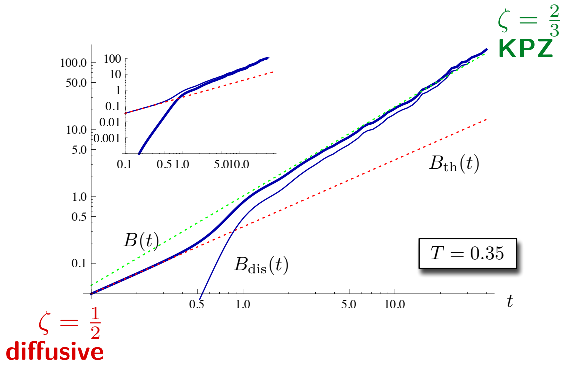

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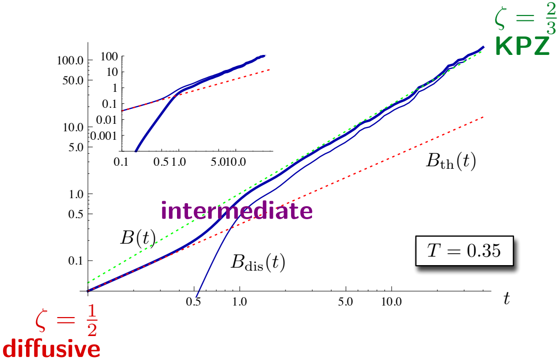

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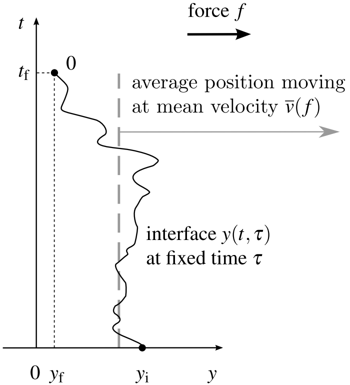
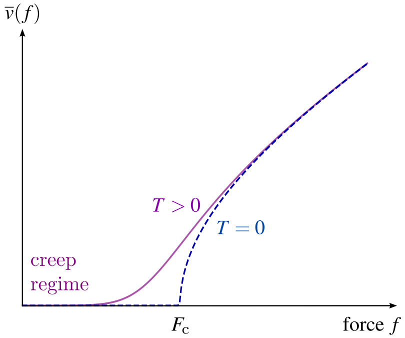
- Interpretation in other '**incarnations**' of the KPZ class
  - ⋆ growth interfaces with $F(t, y)$ = height at (real) time t
  - ⋆ experimental probe of the importance of ξ
  - ⋆ through replicæ: **1D quantum bosons** with softened attractive interaction

Non-linear response at small force





Creep law: non-linear response to small force

$$\text{velocity} \sim \exp\left\{ -\left[\overbrace{\frac{\text{critical force}}{\text{force}}}^{\text{depends on } c,\ D,\ T,\ \xi}\right]^{1/4} \right\}$$

Effective model