Interfaces in random media with short-range correlated disorder

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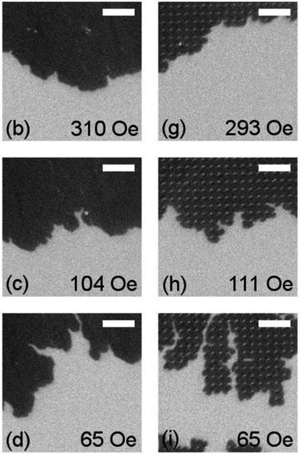
Grenoble – 5th June 2014

1D Interfaces

Interfaces in magnetic films



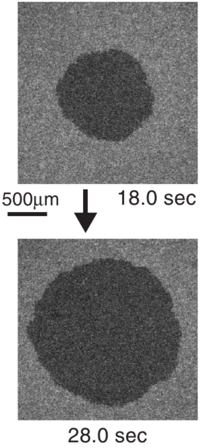
from Metaxas *et al.*

APL **94** 132504 (2009)

Large range of physical scales

Wide spectrum of phenomena

Growth in liquid crystals



from Takeuchi & Sano

PRL **104** 230601 (2010)

Disordered elastic systems

- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\text{el}} = \frac{c}{2}\int dz\,\big(\nabla u(z)\big)^2 \qquad \text{[Short-range]}$$

$$\mathcal{H}^{\text{el}} = \frac{c}{2\pi}\int dz dz' \,\frac{\big(u(z)-u(z')\big)^2}{(z-z')^2} \qquad \text{[Long-range]}$$

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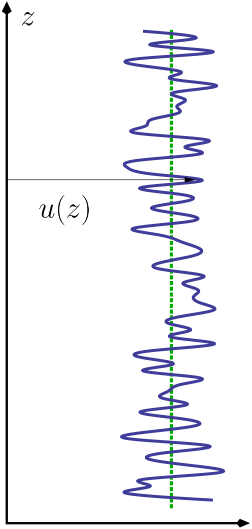

Competition btw “order” and “disorder”

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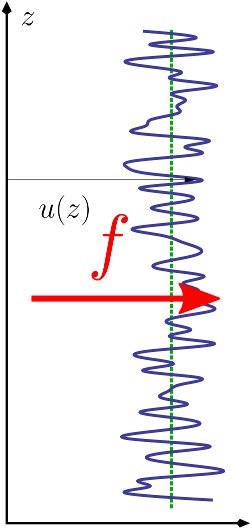
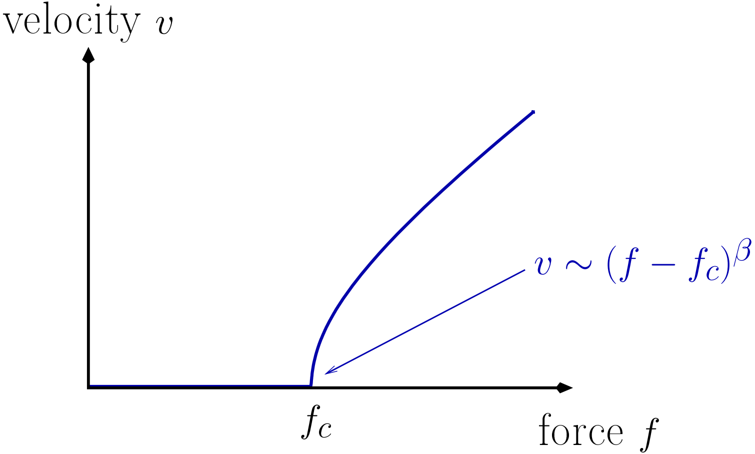
- Force: induces motion of the interface



Competition btw "order" and "disorder"

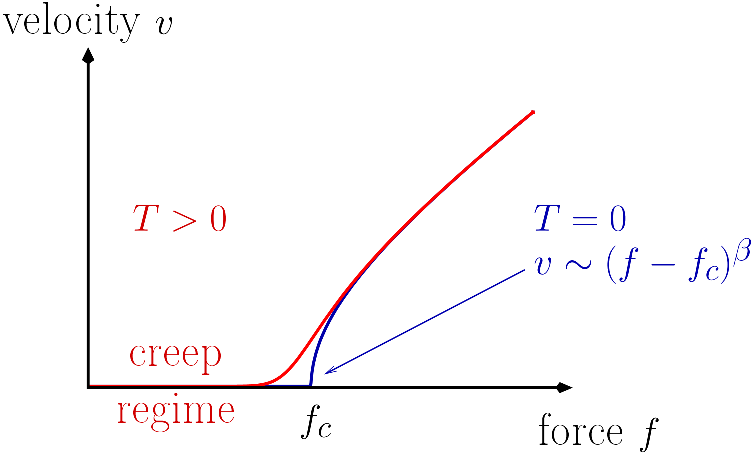
Depinning transition @ zero temperature

threshold force f_c



Depinning transition @ finite temperature

thermal rounding
creep regime



Uncorrelated disorder:

$$\overline{V(z,x)\,V(z',x')} = D\,\delta(z'-z)\delta(x'-x)$$

Correlated disorder on a lengthscale ξ:

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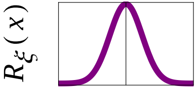

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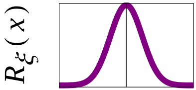
Can ξ play a role at lengthscales $\gg \xi$?



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Study 1D models with correlated disorder $(\xi > 0)$

1. Static properties & creep regime
   short-range elasticity
   $\longrightarrow$ **Identification of lengthscales**
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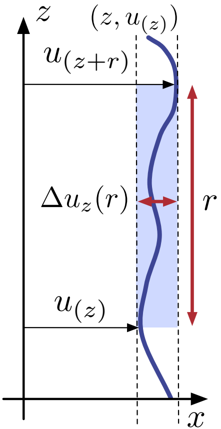
1D Interface in the Directed Polymer (DP) language

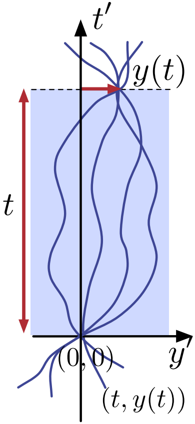
[Step n°1]

- No bubbles
- No overhangs
- Interface lengthscale r

$\updownarrow$

DP 'time' t





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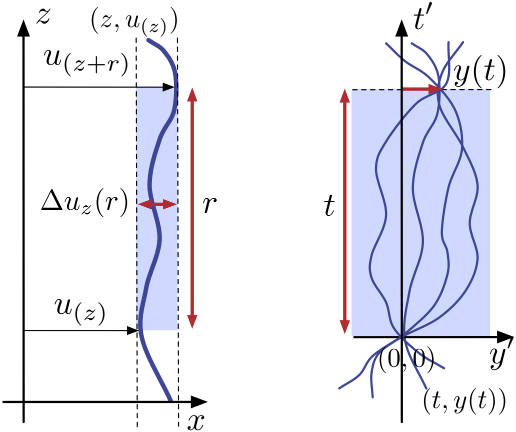

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lengthscale ≡time duration

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Competition btw "order" and "disorder"

- Ingredients up to now:

elastic constant c disorder potential $V(t,y)$ $\overbrace{\text{temperature } T}^{\text{trajectory weight } \propto e^{-\mathcal{H}_V/T}}$

Questions

- Nature of fluctuations
  - ⋆ $V(t,y) \equiv 0$: *diffusive* ($y \sim t^{1/2}$), **Edwards-Wilkinson** (EW)
  - ⋆ $V(t,y) \not\equiv 0$: *super-diffusive* ($y \sim t^{2/3}$), **Kardar-Parisi-Zhang** (KPZ)

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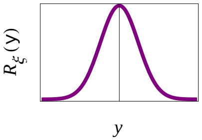
- Role of (experimentally ineluctable) **disorder correlations**?

  zero mean, Gaussian, $\overline{V(t,y)V(t',y')} = D\delta(t'-t)R_\xi(y'-y)$



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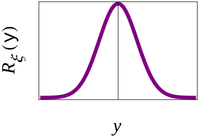

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- Summary of ingredients:

  elastic constant c temperature T disorder | amplitude D, corr. length ξ

Free-energy fluctuations [Step n°2&3]

- Partition function Z_V *vs.* Free-energy F_V

$$Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t')e^{-\frac{1}{T}\mathcal{H}_V[y(t'),t]} \qquad F_V(t,y) = -\frac{1}{T}\log Z_V(t,y)$$

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- **Statistical Tilt Symmetry**

$$F_V(t,y) = \underbrace{c\frac{y^2}{2t} + \frac{T}{2}\log\frac{2\pi Tt}{c}}_{\substack{\text{thermal contribution} \\ F_{V\equiv 0}}} + \underbrace{\bar{F}_V(t,y)}_{\substack{\text{disorder} \\ \text{contribution}}} \qquad \textbf{(STS)}$$

- **Tilted** KPZ equation for $\bar{F}_V(t,y)$

$$\partial_t \bar{F}_V + \frac{y}{t}\partial_y \bar{F}_V = \frac{T}{2c}\partial_y^2 \bar{F}_V - \frac{1}{2c}\left[\partial_y \bar{F}_V\right]^2 + V(t,y)$$

Non-linear, additive noise, $\bar{F}_V(0,y) \equiv 0$: “simple” initial cond.

Known results $@\xi = 0$ $[\Longleftrightarrow T \to \infty \ @\xi > 0]$

- **Central tool**: 2-point correlation function

$$\bar{R}(t, y_2 - y_1) = \overline{\partial_y \bar{F}_V(t, y_1) \partial_y \bar{F}_V(t, y_2)}$$

- **Infinite-'time' limit** (steady state)

$\bar{F}(t = \infty, y)$ distributed as a Brownian Motion

i.e. : $Prob\big[\bar{F}(t = \infty, y)\big]$ Gaussian, of correlator

$$\bar{R}(t = \infty, y) = \widetilde{D}_{\xi=0}\, \delta(y) \quad \text{with} \quad \boxed{\widetilde{D}_{\xi=0} = \frac{cD}{T}}$$

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- **Roughness** function $B(t)$ [variance of end-point fluct.]

$$B(t) = \overline{\langle y(t)^2 \rangle} = \overline{\frac{\int dy\, y^2 Z_V(t,y)}{\int dy\, Z_V(t,y)}}$$

$$\boxed{B(t) = \big[\widetilde{D}_{\xi=0} / c^2\big]^{2/3} t^{4/3}} \quad \text{as } t \to \infty$$

Effective model @$\xi > 0$ & numerical results

$\xi > 0$ not obtained from perturbation of $\xi = 0$

- **Distribution** of free-energy

  scales closely to the $\xi = 0$ case

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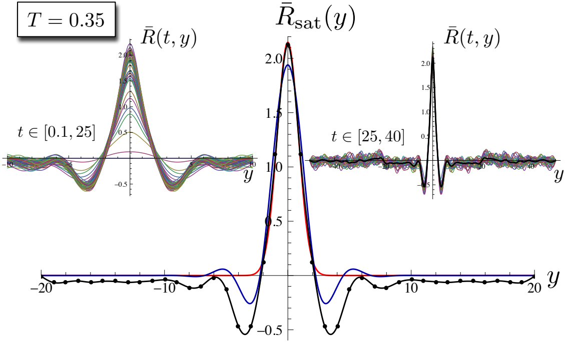
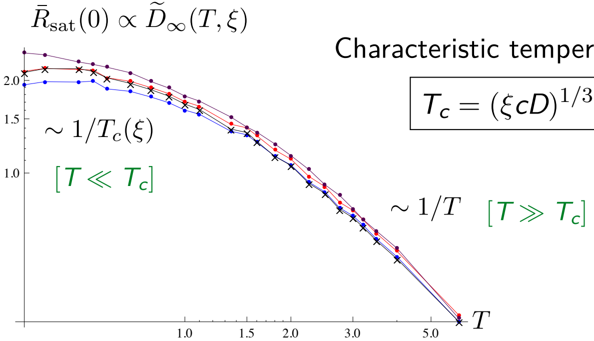
- **Distribution** of free-energy
  scales closely to the $\xi = 0$ case
- **2-point** correlation function of amplitude $\widetilde{D}$

$$\bar{R}(t, y) \simeq \widetilde{D} R_\xi(y) \text{ as } t \to \infty$$



High- and low-temperature regimes



Characteristic temperature

$$T_c = (\xi c D)^{1/3}$$

- (Advanced) **scaling** analysis

$T \ll T_c$	$T \gg T_c$
one optimal trajectory	many trajectories
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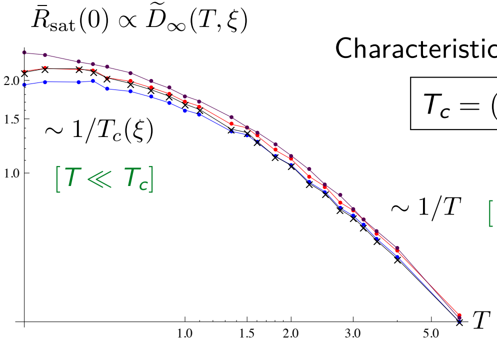

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Lengthscales & dynamics

PRE 87 042406 (2013)

- **Geometry** of interface $\longleftrightarrow$ Directed Polym. **free-energy** fluctuat.
  - ⋆ $T \lesssim T_c$: ξ **plays a role at all lengthscales** $[T_c = (\xi c D)^{1/3}]$
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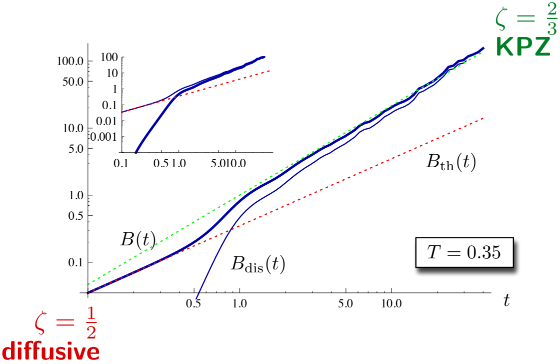

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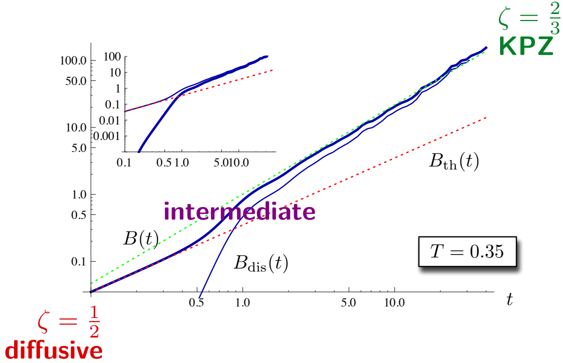

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- **Creep law**: non-linear response to small force

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- Interpretation in other '**incarnations**' of the KPZ class
  - ⋆ growth interfaces with $F(t, y) =$ height at (real) time t
  - ⋆ experimental probe of the importance of ξ
  - ⋆ through replicæ: **1D quantum bosons** with softened attractive interaction

Depinning transition

[following V Démery, L Ponson, A Rosso EPL **105** 34003 (2014)]

Equation of evolution [long-range elasticity]

$$\partial_t u(z,t) = f_{\mathrm{el}}[u(\cdot,t)](z) - \sigma \partial_u V\big(u, u(z,t)\big)$$

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- ⋆ perform a 1$^{\mathrm{st}}$ order **perturbation** in disorder
- ⋆ obtain force(velocity) $\Longrightarrow$ get velocity(force)

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Disorder correlations: [for the force]

$$\overline{\partial_z V(z,x) \partial_z V(z',x')} = \Delta_u(z'-z)\Delta_x(x'-x)$$

Critical force

Result:

$$f_c = \frac{\sigma^2 \widetilde{\Delta}_x(0)}{4\pi c} \int dk_u \, |k_u| \widetilde{\Delta}_u(k_u) \qquad (\sigma \to 0)$$

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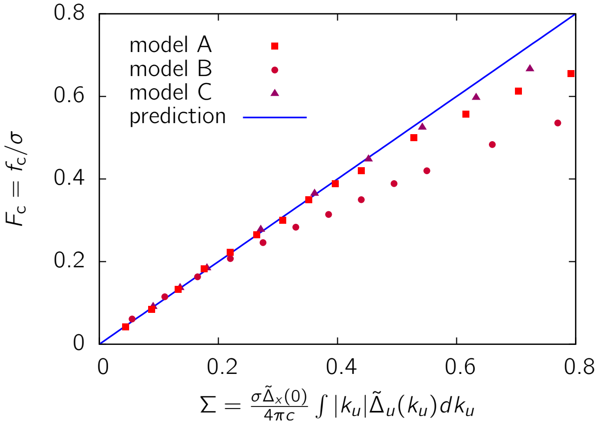

Thank you for your attention!
Bon appétit!

References:

- Elisabeth Agoritsas, Vivien Lecomte & Thierry Giamarchi:
  - Phys. Rev. E **87** 062405 (2013)
  - Phys. Rev. E **87** 042406 (2013)
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- Vincent Démery, Vivien Lecomte & Alberto Rosso:
  - J. Stat. Mech. **P03009** (2014)