Interfaces in disordered media

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Grenoble – LIPhy internal seminar – 20 September 2018

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Outline

1. Examples
   - Systems
   - Questions
2. A panorama of results
   - Zero temperature
   - Small temperature
3. Role of hidden degrees of freedom
   - Examples
   - Results
4. Open questions and perspectives
   - Active matter
   - Soft matter

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T. Halpin-Healy, Y.-C. Zhang / Physics Reports 254 (1995) 215–414

1. Introduction

Consider a wheat field of dark golden hue and densely planted in level ground, being roughly rectangular in shape, but rather large in extent, and stretching lazily toward the distant horizon. On a cool, but calm August evening, with nary a breeze about, the edge of the field is ignited, in preparation for leaving the soil fallow the following season. The propagating fire front, initially straight by virtue of its birth along the edge, evolves in a kinetic, violent fashion and heads mercilously into the bulk of the field. Burning shafts of wheat communicate the conflagaration locally to their neighbors, and the narrow, bright, and tortuously shaped fire line, an interface separating the blackened region from the portion of the field soon to be consumed, becomes increasingly rough as random elements, such as local inhomogeneities in the moisture content or density of the wheat, begin to have a large scale cumulative effect.

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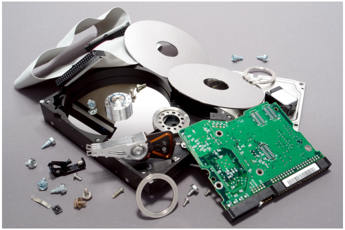
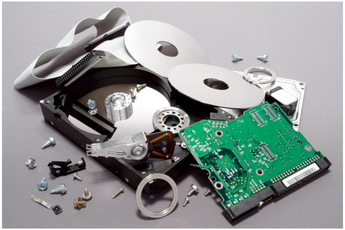
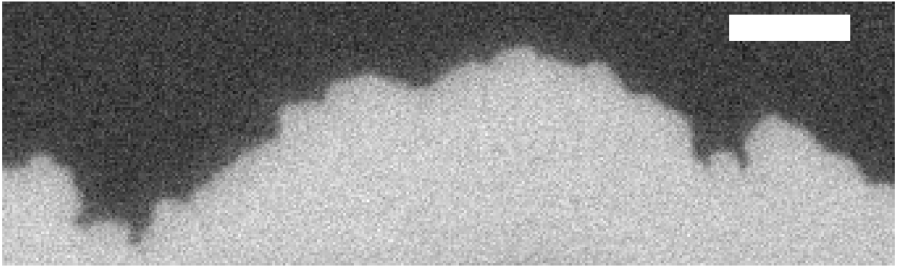
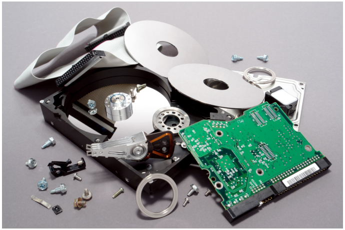
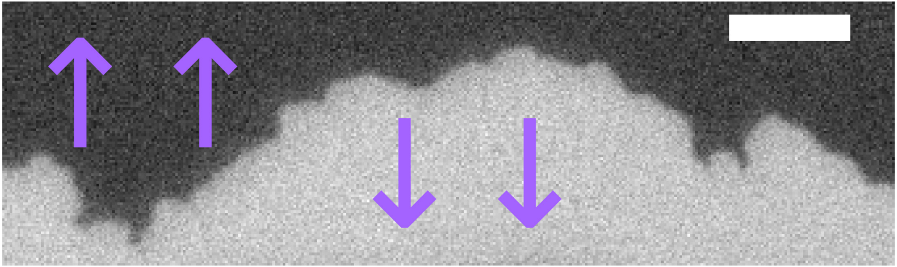






[Metaxas *et al.* PRL **99** 217208 (2007)]





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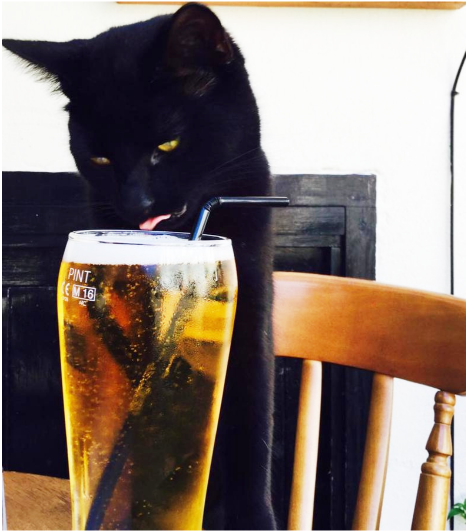
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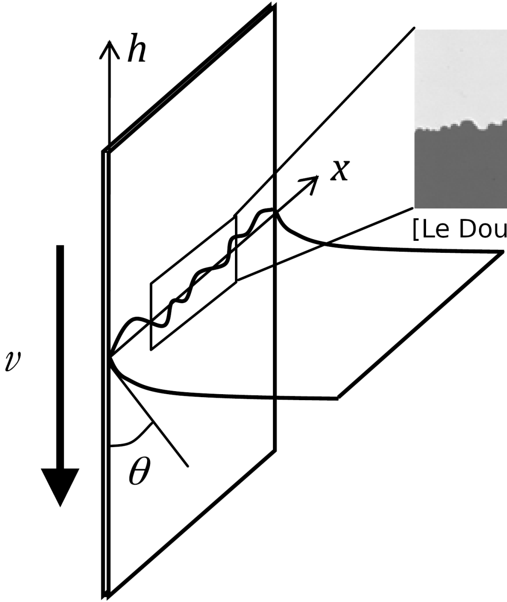
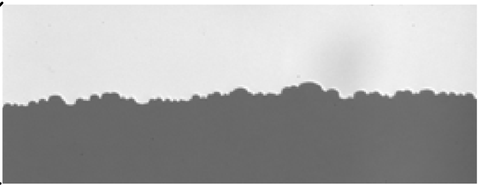
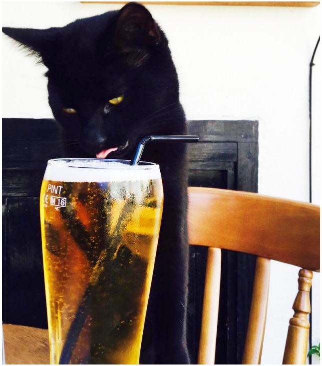


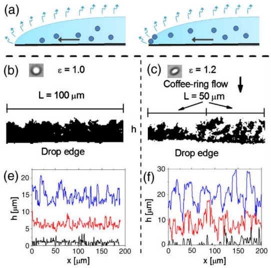


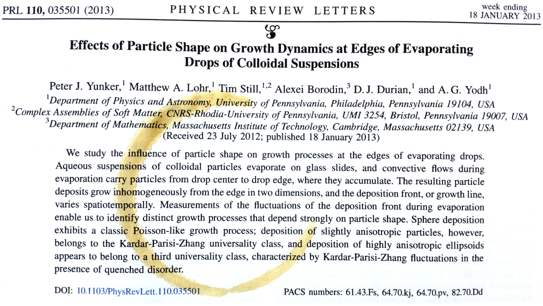
[Le Doussal, Wiese, Moulinet, Rolley, EPL **87** 56001 (2009)]



Front of evaporation/imbibition

PRL **110,** 035501 (2013)





PRL **110,** 035501 (2013) PHYSICAL REVIEW LETTERS week ending 18 JANUARY 2013

Effects of Particle Shape on Growth Dynamics at Edges of Evaporating Drops of Colloidal Suspensions

Peter J. Yunker,[1] Matthew A. Lohr,[1] Tim Still,[1,2] Alexei Borodin,[3] D. J. Durian,[1] and A. G. Yodh[1]
[1]*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*
[2]*Complex Assemblies of Soft Matter, CNRS-Rhodia-University of Pennsylvania, UMI 3254, Bristol, Pennsylvania 19007, USA*
[3]*Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
(Received 23 July 2012; published 18 January 2013)

We study the influence of particle shape on growth processes at the edges of evaporating drops. Aqueous suspensions of colloidal particles evaporate on glass slides, and convective flows during evaporation carry particles from drop center to drop edge, where they accumulate. The resulting particle deposits grow inhomogeneously from the edge in two dimensions, and the deposition front, or growth line, varies spatiotemporally. Measurements of the fluctuations of the deposition front during evaporation enable us to identify distinct growth processes that depend strongly on particle shape. Sphere deposition exhibits a classic Poisson-like growth process; deposition of slightly anisotropic particles, however, belongs to the Kardar-Parisi-Zhang universality class, and deposition of highly anisotropic ellipsoids appears to belong to a third universality class, characterized by Kardar-Parisi-Zhang fluctuations in the presence of quenched disorder.

DOI: 10.1103/PhysRevLett.110.035501 PACS numbers: 61.43.Fs, 64.70.kj, 64.70.pv, 82.70.Dd

Front of evaporation/imbibition

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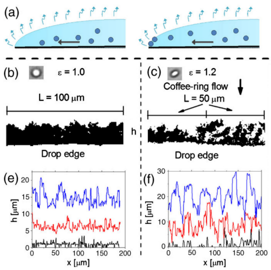

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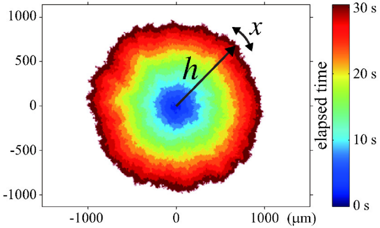
Front between turbulent modes in liquid crystals

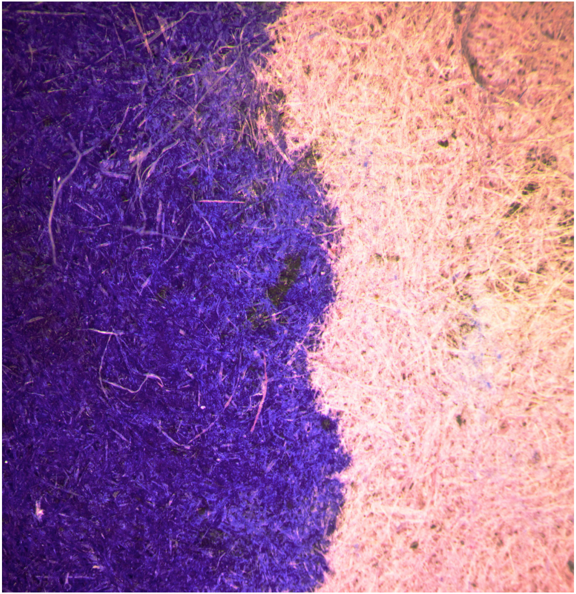


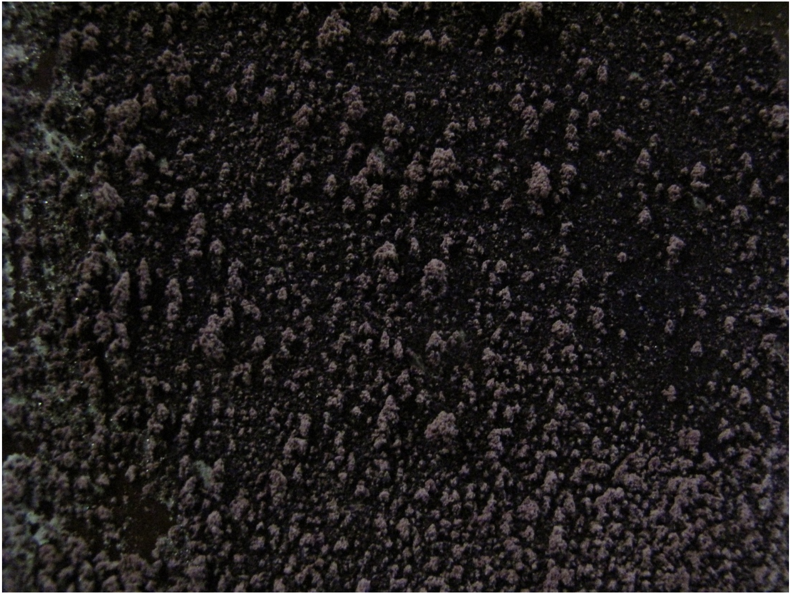
J Stat Phys (2015) 160:794–814
DOI 10.1007/s10955-015-1282-1

A KPZ Cocktail-Shaken, not Stirred...

Toasting 30 Years of Kinetically Roughened Surfaces

Timothy Halpin-Healy[1] · Kazumasa A. Takeuchi[2,3]



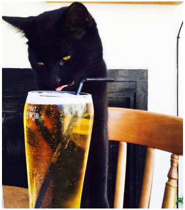


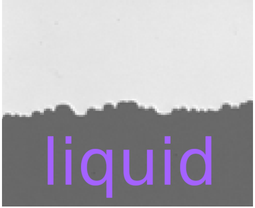
Large range of physical scales

Wide spectrum of phenomena

Questions:

- What are the main physical ingredients?
- How to characterise the geometry and the dynamics?



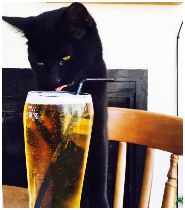


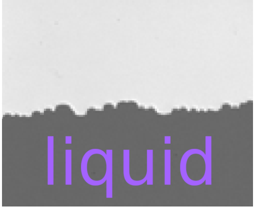
Order: surface tension

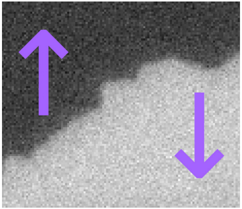
Disorder: substrate impurities

Drive: imposed ⟨liquid level⟩

Noise: negligible







Order: surface tension

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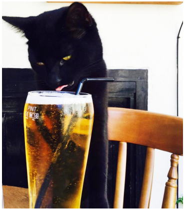
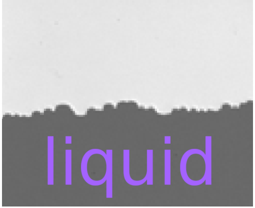
Order: energetic cost of interface

Disorder: magnetic impurities

Drive: external magnetic field

Noise: thermal

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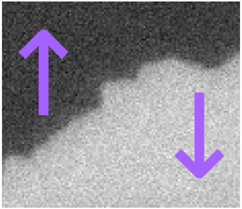

Order: energetic cost of interface

Disorder: magnetic impurities

Drive: external magnetic field

Noise: thermal

Order: convex patches burn faster

Disorder: inhomogeneities

Drive: instability of unburnt grass

Noise: turbulence in the air

Large range of physical scales

Wide spectrum of phenomena

Questions:

- What are the main physical ingredients?
- How to characterise the geometry and the dynamics?

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- What are the main physical ingredients?
  - Competition between
    order (tends to align)
    quenched **disorder** (tends to deform)
  - **Noise** (space and time fluctuating force)
  - **Drive** (external force or internal instability)

- How to characterise the geometry and the dynamics?

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  - **Noise** (space and time fluctuating force)
  - **Drive** (external force or internal instability)
- When is disorder *relevant*?
- How to characterise the geometry and the dynamics?

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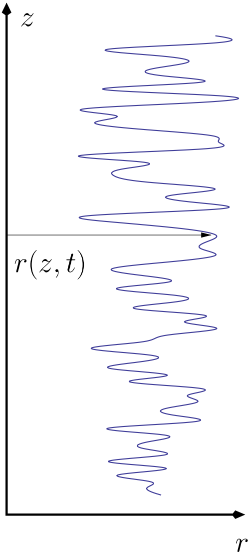
   - Questions
2. A panorama of results
   - Zero temperature
   - Small temperature


z
$r(z,t)$
r

- **Geometry**:
  - Roughness function B

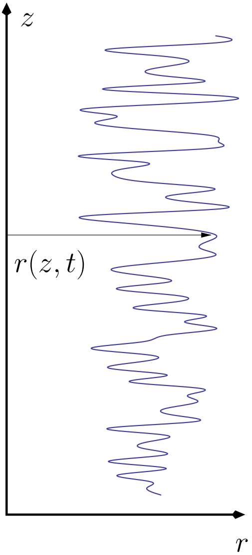
$$B(z,t) = \overline{\langle [r(z_1+z,t_1+t) - r(z_1,t_1)]^2 \rangle}$$

$\langle \dots \rangle$ = thermal average

$\overline{\dots}$ = disorder average

  - Roughness exponent ζ

$$B(z,0) \sim z^{2\zeta} \qquad (z \to \infty)$$



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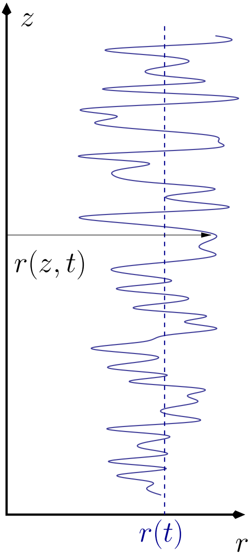

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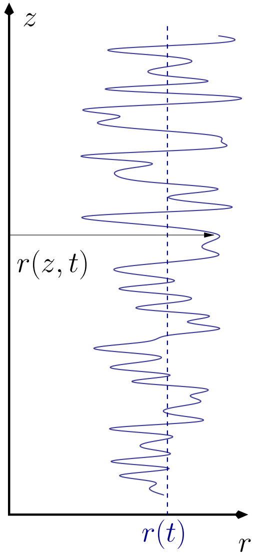
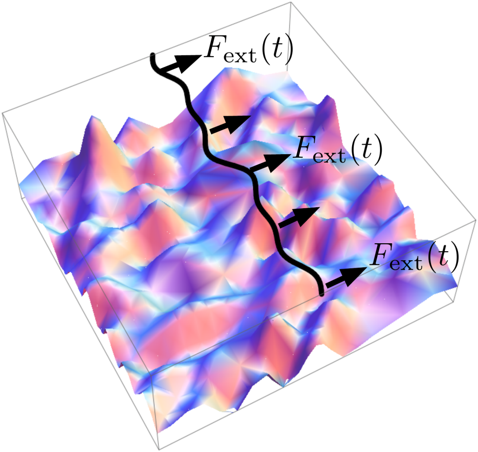
- **Dynamics**:
  - Velocity-force characteristic

$$v(f) = \overline{\langle \partial_t r(t) \rangle} \qquad (t \to \infty)$$

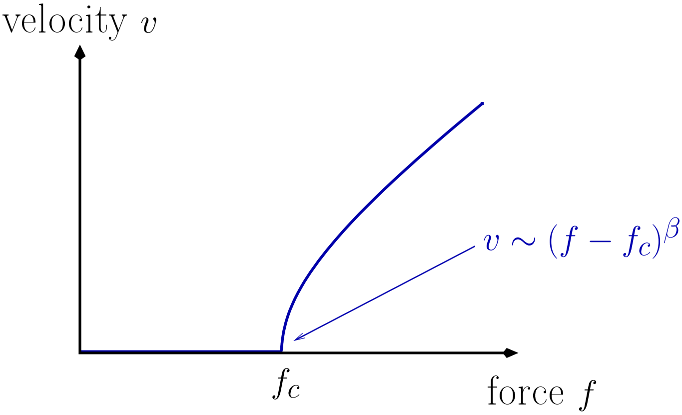


The velocity-force characteristic $v(f)$



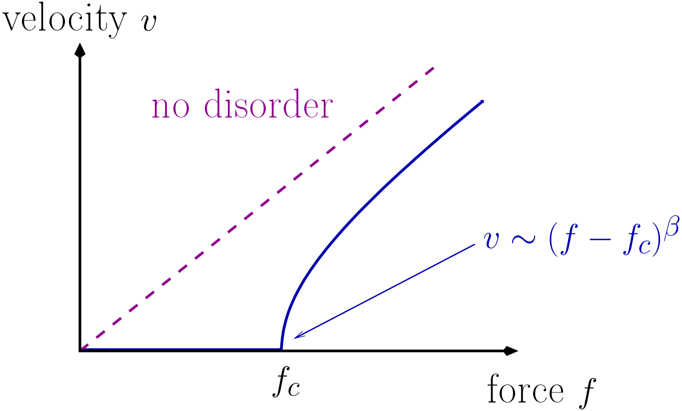
Schematic representation of the protocol.

Depinning transition @ zero temperature



Criticality at a threshold force f_c [*non-equilibrium* phase transition]

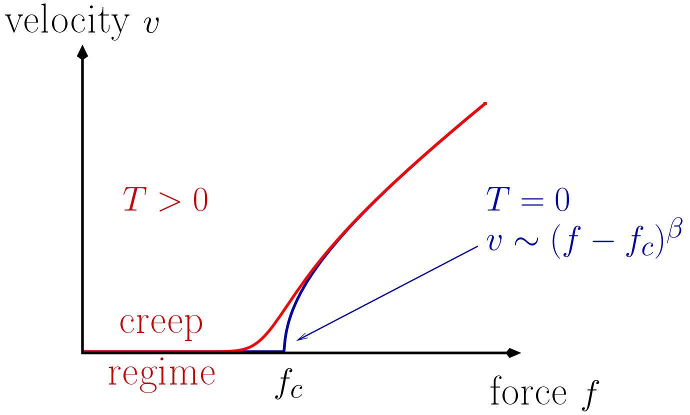
Depinning transition & finite temperature



Criticality at a threshold force f_c [*non-equilibrium* phase transition]

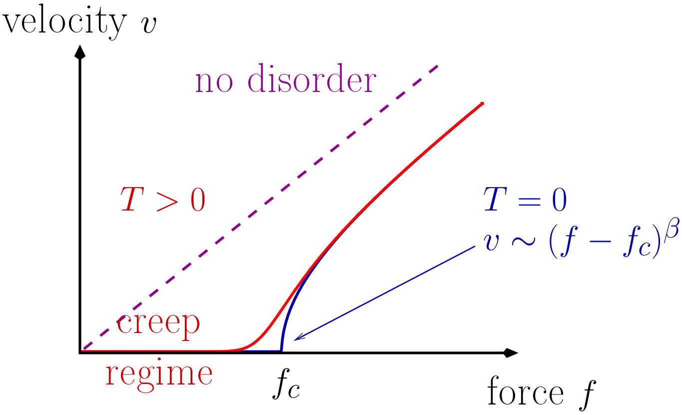
Disorder is **relevant**

Depinning transition & finite temperature



Creep law: $v(f) \overset{f\to 0}{\sim} e^{-\frac{U_c}{T}(f_c/f)^\mu}$ [**Highly non-linear** response]

Depinning transition & finite temperature



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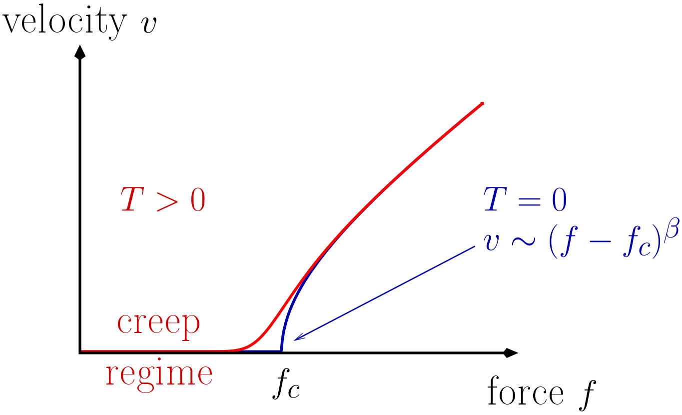

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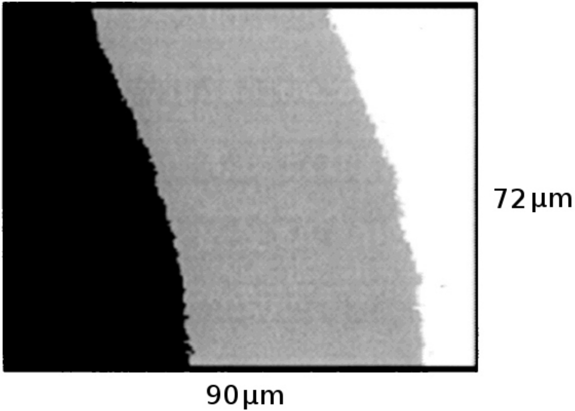
Disorder is **relevant**

$$v(f) \sim \exp\left[-\frac{U_c}{T}\left(\frac{f_c}{f}\right)^{\mu}\right] \qquad \text{(creep law)}$$



$\mu = 1/4$ here

[Lemerle *et al.*, PRL **80** 849 (1998)]

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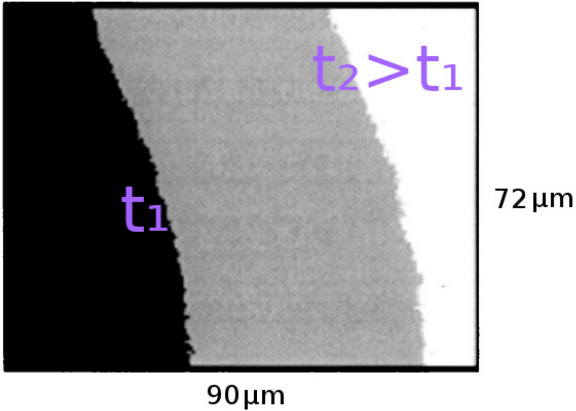

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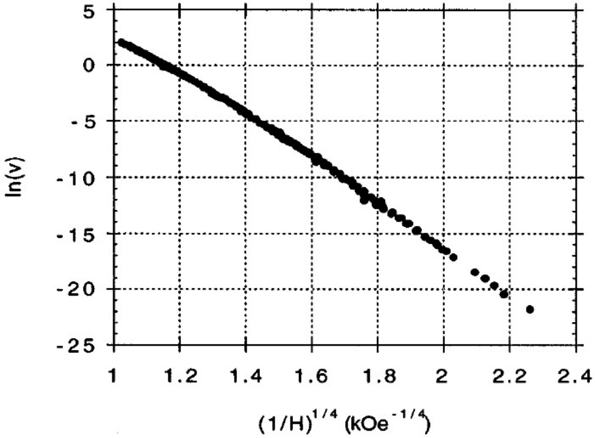

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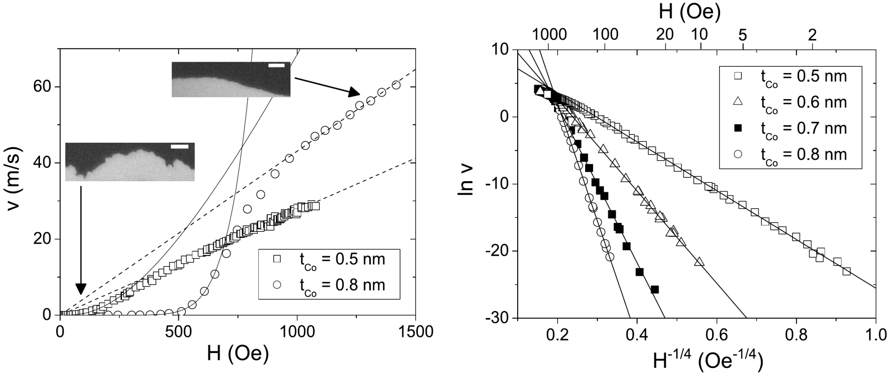

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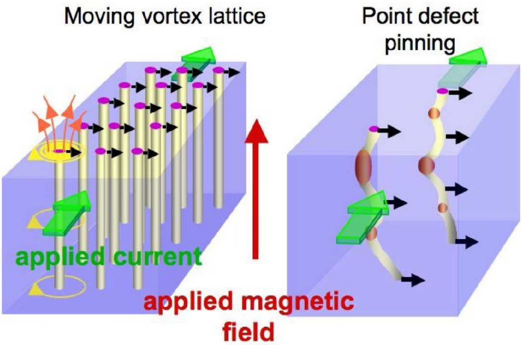
A remarkable consequence



[Matti Irjala]

Without disorder, type II superconductors would dissipate Ohmically.

A simple picture of depinning?

Disordered elastic systems

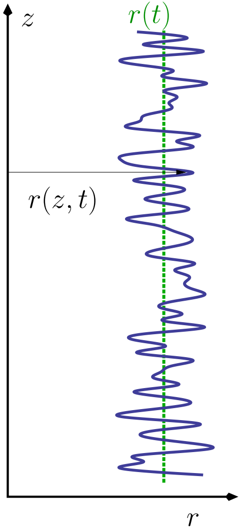
- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\mathrm{el}} = \frac{c}{2}\int dz\, \left(\nabla r(z)\right)^2 \qquad \textbf{[Short-range]}$$

$$\mathcal{H}^{\mathrm{el}} = \frac{c}{2\pi}\int dz dz' \,\frac{\left(r(z)-r(z')\right)^2}{(z-z')^2} \qquad \text{[Long-range]}$$

- Disorder: tends to deform it

$$\mathcal{H}^{\mathrm{dis}}_V = \int dz\; V\big(z, r(z)\big)$$



Competition btw “order” and “disorder”

Disordered elastic systems

- Elasticity: tends to flatten the interface

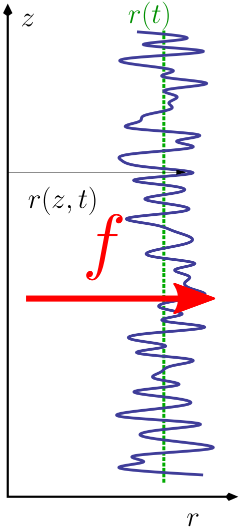
$$\mathcal{H}^{\text{el}} = \frac{c}{2}\int dz\,\left(\nabla r(z)\right)^2 \qquad \textbf{[Short-range]}$$

$$\mathcal{H}^{\text{el}} = \frac{c}{2\pi}\int dz dz' \,\frac{\left(r(z)-r(z')\right)^2}{(z-z')^2} \qquad \text{[Long-range]}$$

- Disorder: tends to deform it

$$\mathcal{H}_V^{\text{dis}} = \int dz\; V(z, r(z))$$

- Force: induces motion of the interface



Competition btw “order” and “disorder”

Some known results

1. Huge variety of physical systems and theoretical approaches
   - ⋆ Elastic manifolds (lines, membranes, interfaces); periodic (vortex lattices); growth interfaces (aggregation, wetting)
   - ⋆ Methods: field theory, renormalisation group, scaling analysis, exactly solvable models, replica methods
   - ⋆ Reviews: Halpin-Healy&Zhang; Blatter&*al.*; Quastel; Corwin
2. Nature of fluctuations in **dimension 1+1** (elastic line)
   - ⋆ No disorder ($V(z,r) \equiv 0$):
     diffusive ($u \sim z^{1/2}$), **Edwards-Wilkinson** (EW)
   - ⋆ Disorder ($V(z,r) \not\equiv 0$):
     super-diffusive ($u \sim z^{2/3}$), **Kardar-Parisi-Zhang** (KPZ)

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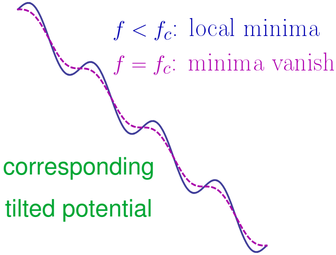
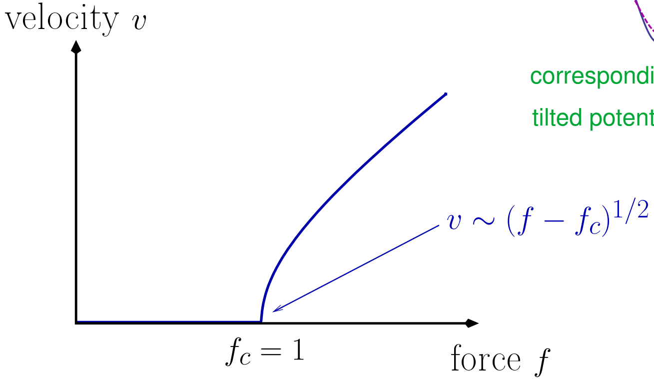
Disorder is **always** relevant

Depinning @ zero temperature

Effective model for the mean interface position $r(t)$

$$\alpha \partial_t r = \underbrace{f - \cos \kappa r}_{\text{total force}}$$

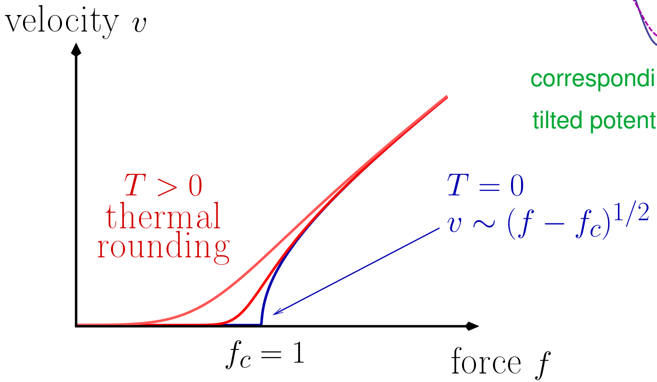




Depinning @ finite temperature

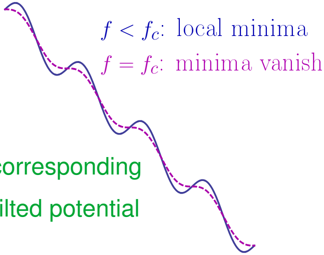
Effective model for the mean interface position $r(t)$

$$\alpha\partial_t r = \underbrace{f - \cos\kappa r}_{\text{total force}} + \eta$$



corresponding

tilted potential



Depinning @ finite temperature

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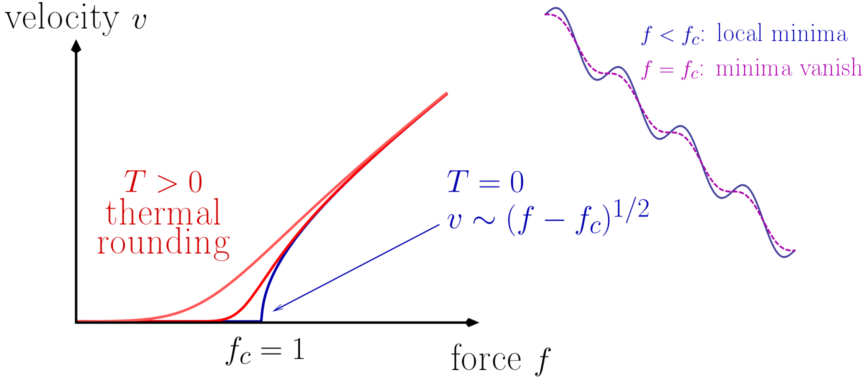
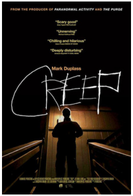


- Depinning: **ok**, but "mean-field" depinning exponent $1/2$
- Creep: **not ok** (linear response at $f \ll f_c$ instead of creep law)

A simple picture of creep?

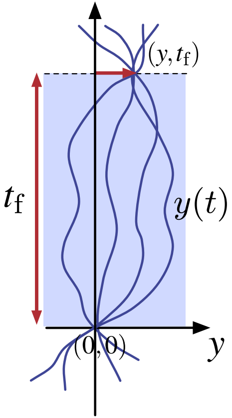


1D interface in the Directed Polymer (DP) language

- No bubbles
- No overhangs
- Interface lengthscale z

$\updownarrow$

DP 'time' t_f



working at fixed 'time' t_f $\Longleftrightarrow$
integration of fluctuations at scales smaller than t_f

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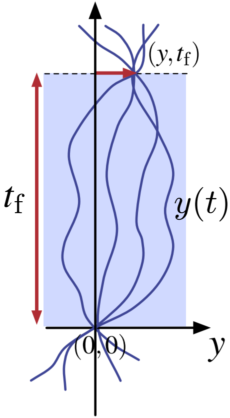

working at fixed 'time' t_f $\Longleftrightarrow$
integration of fluctuations at scales smaller than t_f

lengthscale $\equiv$ time duration

Free-energy fluctuations of trajectories $(0,0) \rightsquigarrow (t,y)$

Time evolution as "renormalisation"

- Partition function Z_V *vs.* Free-energy F_V

$$Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t')\mathrm{e}^{-\frac{1}{T}\mathcal{H}_V[y(t'),t]} \qquad F_V(t,y) = -\frac{1}{T}\log Z_V(t,y)$$

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- **Stochastic Heat Equation** (Feynman-Kac formula)

$$\partial_t Z_V = \Big[\frac{T}{2c}\partial_y^2 - \frac{1}{T}V(t,y)\Big] Z_V(t,y) \qquad \textbf{(SHE)}$$

  Linear, multiplicative noise, reversible

- **Kardar-Parisi-Zhang** equation

$$\partial_t F_V = \frac{T}{2c}\partial_y^2 F_V - \frac{1}{2c}\big[\partial_y F_V\big]^2 + V(t,y) \qquad \textbf{(KPZ)}$$

  Non-linear, additive noise, non-reversible

Free-energy fluctuations of trajectories $(0,0) \rightsquigarrow (t,y)$

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Non-linear, additive noise

[$F_V(t,y) \equiv$ interface height at position y and time t]

Statistical tilt symmetry

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- **Statistical Tilt Symmetry**

$$F_V(t,y) = \underbrace{c\frac{y^2}{2t} + \frac{T}{2}\log\frac{2\pi Tt}{c}}_{\substack{\text{thermal contribution} \\ F_{V\equiv 0}}} + \underbrace{\bar{F}_V(t,y)}_{\substack{\text{disorder} \\ \text{contribution}}} \qquad \textbf{(STS)}$$

Known result

- **Infinite-time limit** $t_{\mathrm{f}} \to \infty$ (steady state)

  $\bar{F}(t_{\mathrm{f}} = \infty, y)$ distributed as a Brownian motion along y

  i.e.: $\mathbb{P}\big[\bar{F}(t_{\mathrm{f}} = \infty, y)\big]$ Gaussian, of correlator

$$\overline{\big[\bar{F}(t_{\mathrm{f}} = \infty, y) - \bar{F}(t_{\mathrm{f}} = \infty, y')\big]^2} = \widetilde{D}\,|y - y'|$$

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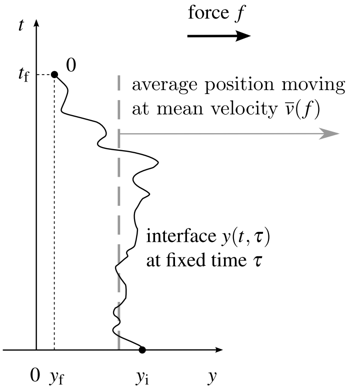
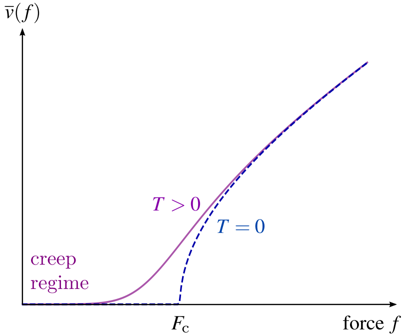
- Rescaling of the disorder free-energy $\bar{F}(t_{\rm f} = \infty, a\hat{y})$

$$\bar{F}(t_{\rm f} = \infty, a\hat{y}) \overset{d}{=} a^{1/2}\widetilde{D}^{1/2} \underbrace{\hat{F}(t_{\rm f} = \infty, \hat{y})}_{\widetilde{D}=1}$$

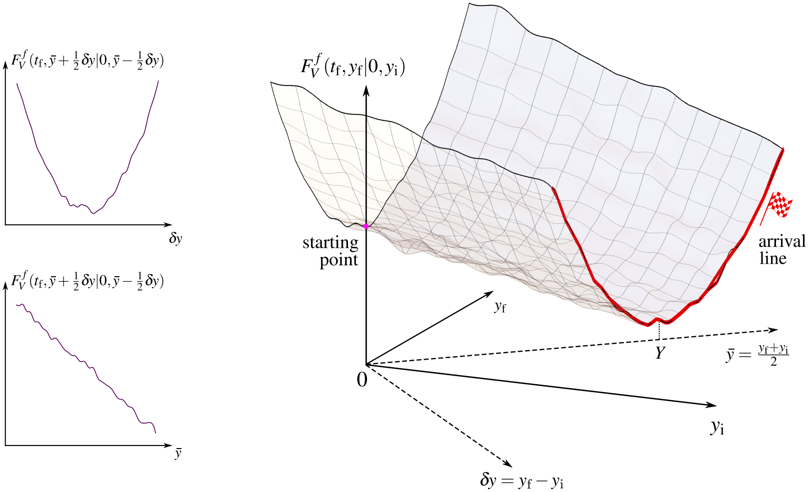
Non-linear response at small force





[Elisabeth Agoritsas, Reinaldo García-García, VL, Lev Truskinovsky and Damien Vandembroucq, J. Stat. Phys. **164** 1394 (2016)]

Effective model



Mean velocity $\longleftrightarrow$ Mean First Passage Time problem (MFPT)

Effective model

Mean velocity $\longleftrightarrow$ Mean First Passage Time problem (MFPT)

- Effective model at fixed $t_{\rm f}$: quasistatic dynamics
  - ⋆ motion of a segment of length $t_{\rm f}$
  - ⋆ extremities $y_{\rm i}$ and $y_{\rm f}$ follow Langevin dynamics
  - ⋆ forces derive from $F_V^f(t_{\rm f}, y_{\rm f}|t_{\rm i}, y_{\rm i})$
  - ⋆ exact at $f = 0$
- Optimisation over $t_{\rm f}$ at fixed f
  - ⋆ optimal $t_{\rm f}$ yielding the avalanche size at fixed f
  - ⋆ saddle-point argument after rescaling
  - ⋆ yields the creep law

$$\text{velocity} \sim \exp\Big\{ - \Big[\frac{\text{critical force}}{\text{force}}\Big]^{1/4} \Big\}$$

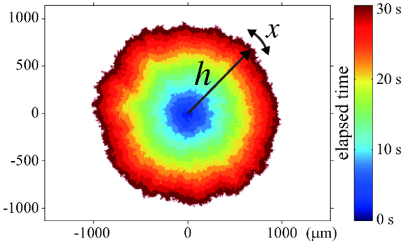
⋆ creep exponent $\frac{1}{4}$ related to the KPZ exponent $\frac{2}{3}$

Geometry: the roughness $B(z)$

Roughness function $B(z)$ (now at equal times)

$$B(z) = \overline{\left\langle \left[r(z_1 + z, t) - r(z_1, t) \right]^2 \right\rangle} \sim z^{2\zeta} \qquad (z \to \infty)$$

Front between turbulent modes in liquid crystals



J Stat Phys (2015) 160:794–814
DOI 10.1007/s10955-015-1282-1

A KPZ Cocktail-Shaken, not Stirred...

Toasting 30 Years of Kinetically Roughened Surfaces

Timothy Halpin-Healy[1] · Kazumasa A. Takeuchi[2,3]

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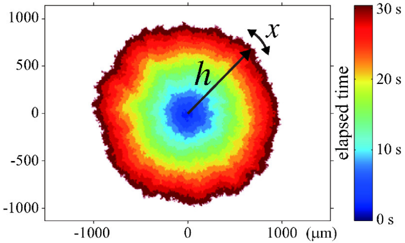

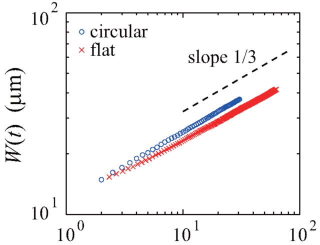

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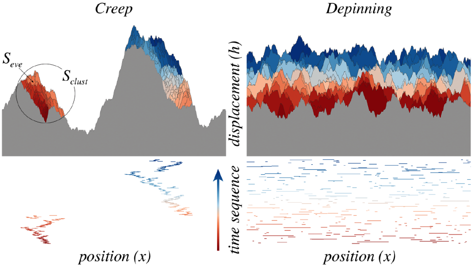
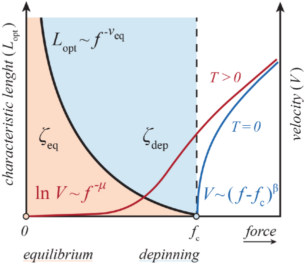
At non-zero f:



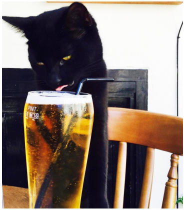


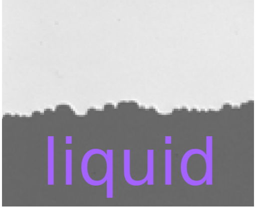
[Ezequiel Ferrero, Laura Foini, Thierry Giamarchi, Alejandro Kolton, Alberto Rosso,PRL **118** 147208 (2017)]

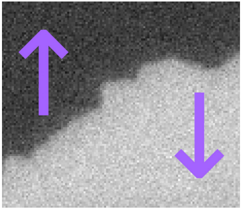
As f increases, ζ moves from $\zeta_{\text{eq}} = \frac{2}{3}$ to $\zeta_{\text{dep}} \approx 1.15$

Outline

1. Examples
   - Systems
   - Questions
2. A panorama of results
   - Zero temperature: the depinning transition
   - Small temperature: creep and thermal rounding
3. Role of hidden degrees of freedom
   - Examples
   - Results



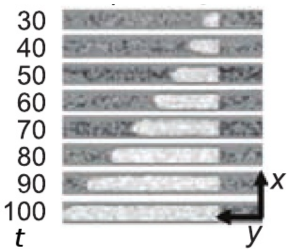

liquid



Is the knowledge of $r(z)$ sufficient?

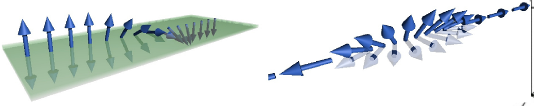
$\rightarrow$ Have a look to the dynamics in simple examples.

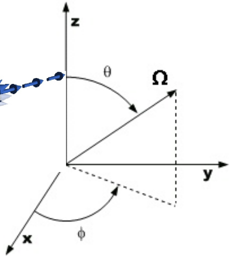
Spintronics



from Yamanouchi *et al.*, Science **317** 1726 (2007)

Bulk model ⇝ effective description





from Tatara *et al.*, J. Phys. Soc. Jap **77** 031003 (2008)

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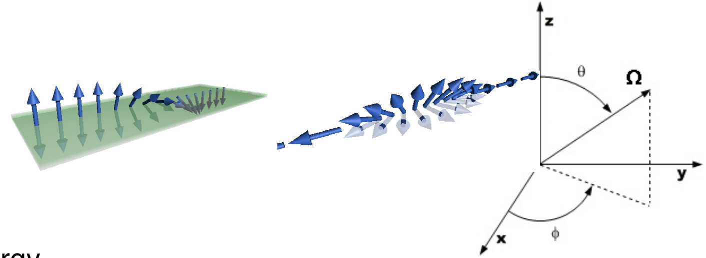


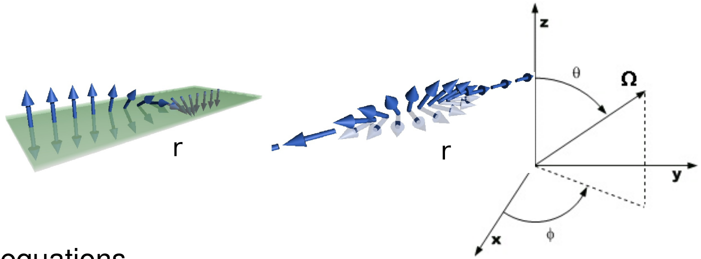
- Bulk energy

$$E = \int d^d x \left\{ J \left[(\nabla\theta)^2 + \sin^2\theta (\nabla\phi)^2 \right] + K \sin^2\theta + K_\perp \sin^2\theta \cos^2\phi \right\}$$

- Equation of motion (Landau-Lifshitz-Gilbert)

$$\partial_t \Omega = \Omega \times \left(\frac{\delta E}{\delta \Omega} + f + \eta \right) - \Omega \times \left(\alpha \partial_t \Omega \right)$$

Bulk model $\rightsquigarrow$ effective description

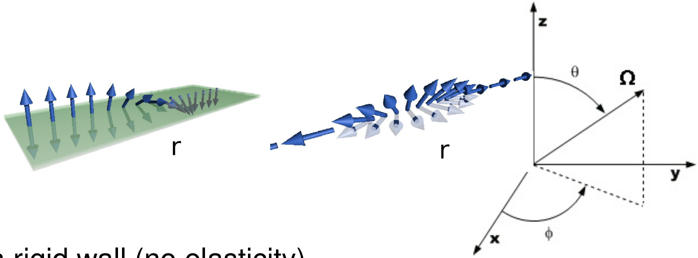


- Effective equations

$$\begin{aligned} \alpha\partial_t r - \partial_t\phi &= J(\nabla r)^2 + F_{\text{pinning}} + f_{\text{ext}} + \eta_1 \\ \underbrace{\alpha\partial_t\phi + \partial_t r}_{\text{damping}} &= \underbrace{J(\nabla\phi)^2 - \frac{1}{2}K_\perp \sin 2\phi}_{\text{forces}} \; \underbrace{+\, \eta_2}_{\text{thermal noise}} \end{aligned}$$

- Further simplification:
  Model reduction (from many to few degrees of freedom)

Bulk model ⇝ effective description



- Case of a rigid wall (no elasticity)

$$\alpha\partial_t r - \partial_t\phi = \overbrace{-\cos\kappa r}^{\text{pinning}} + \overbrace{f}^{\text{external}} + \eta_1$$

$$\alpha\partial_t\phi + \partial_t r = -\frac{1}{2}K_\perp \sin 2\phi + \eta_2$$

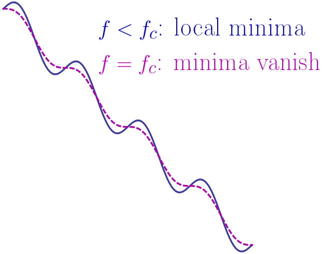
- Effective model: collective degrees of freedom

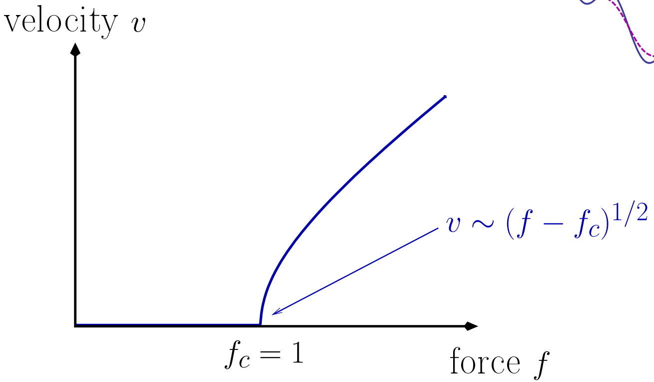
Position $r(t)$ coupled to phase $\phi(t)$

Depinning @ zero temperature

(1st case) Large $K_\perp$: ϕ decouples from r

$$\alpha \partial_t r = \underbrace{f - \cos \kappa r}_{\text{total force}}$$





Depinning @ finite temperature

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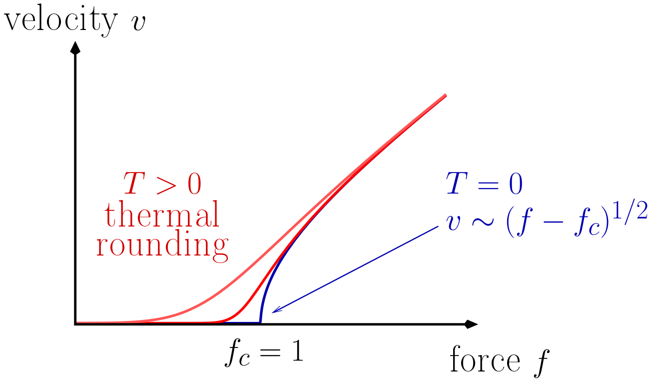
$$\alpha\partial_t r = \underbrace{f - \cos\kappa r}_{\text{total force}} + \eta$$



Depinning @ zero temperature

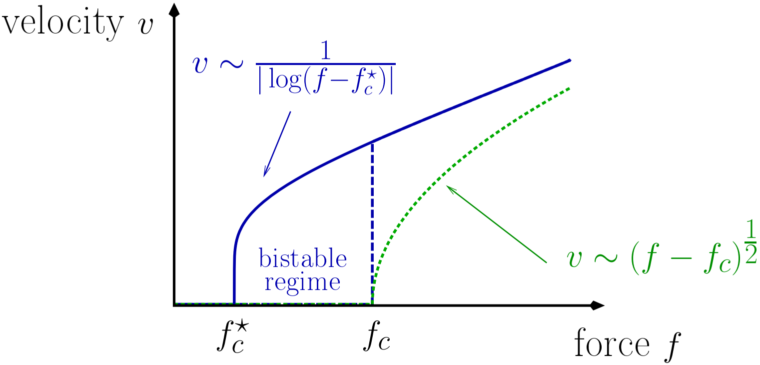
(2nd case) Small $K_\perp$: ϕ matters

$$\alpha\partial_t r - \partial_t\phi = f - \cos\kappa r$$
$$\alpha\partial_t \phi + \partial_t r = -\frac{1}{2}K_\perp \sin 2\phi$$

Depinning @ zero temperature

(2nd case) Small $K_\perp$: ϕ matters

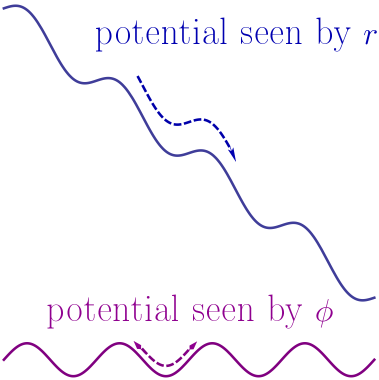
- Dramatic change in the depinning law: $v \sim \frac{1}{|\log(f - f_c^\star)|}$



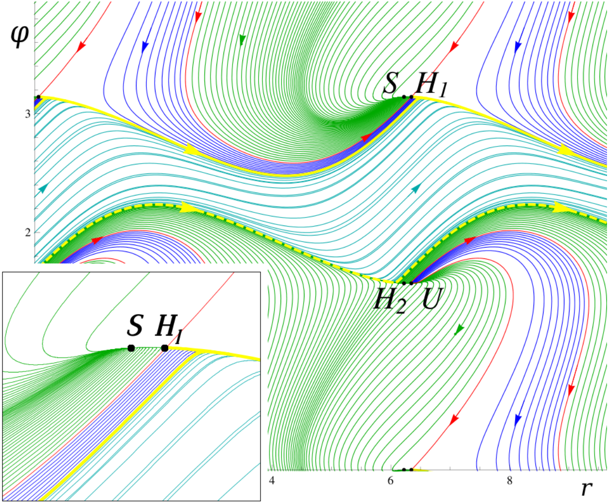
- Depinning at lower critical force: $f_c^\star < f_c$
- Bistability

Physical interpretation

In the bistable regime $f_c^\star < f < f_c$: ϕ helps r to cross barriers



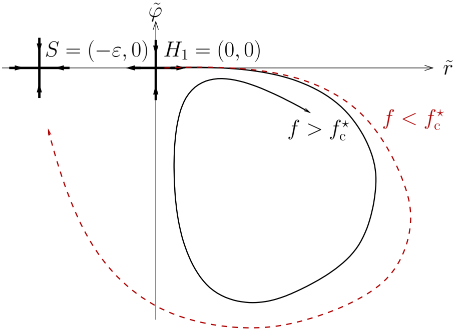
Phase space



In the bistable regime ($f_c^\star < f < f_c$)

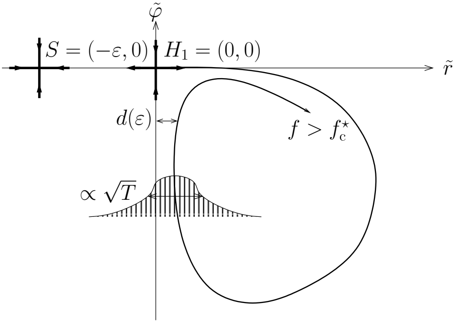
Phase space

Homoclinic bifurcation: $(\epsilon \propto f_c - f)$



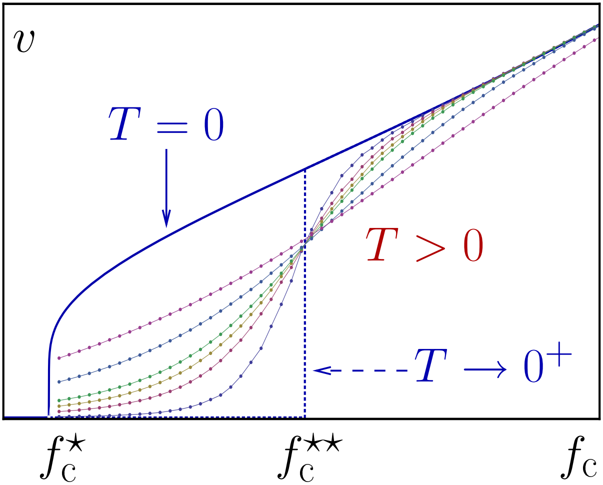
Phase space: $T > 0$

Homoclinic bifurcation with noise: $(\epsilon \propto f_c - f)$



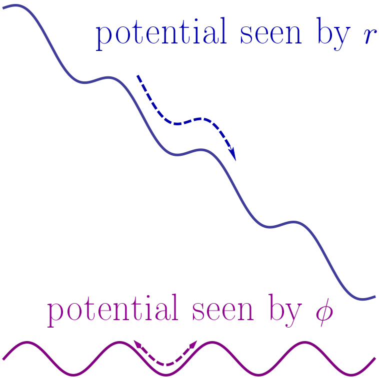
$$\text{escape time} \sim \underbrace{\exp\left(\frac{\epsilon^3}{T}\right)}_{\text{Arrhenius}} \underbrace{\exp\left(-\frac{A}{T}d(\epsilon)^2\right)}_{\text{Trapping probability}}$$

Finite temperature



Force-velocity characteristics

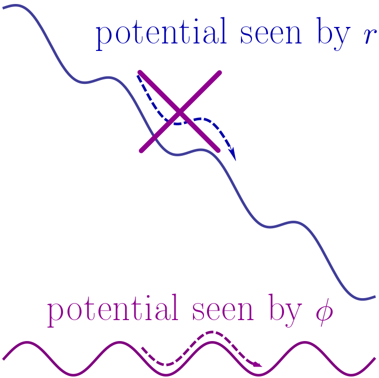
This is not the end of the story



The phase ϕ plays the role of inertia: helps to cross barriers

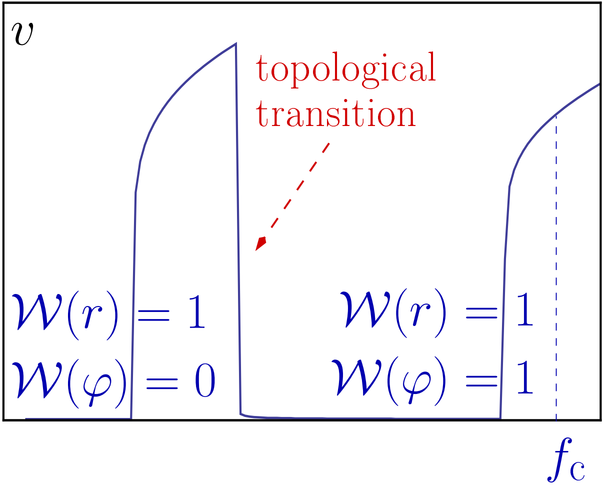
This is not the end of the story

(3[rd] case) Even smaller $K_\perp$



inertia is unbounded whereas ϕ is bounded and periodic

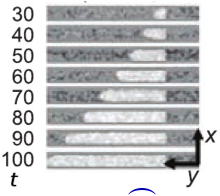
Topological transition



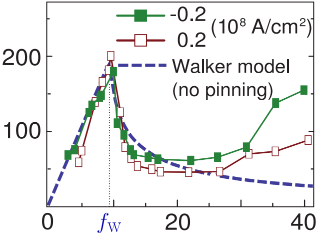
Successive regimes characterised by winding numbers $\mathcal{W}$

Experiment (i)

SPINTRONICS



experiment from Parkin *et al.*, Science **320** 190 (2008)

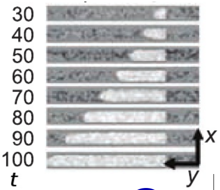


DW velocity (m/s)

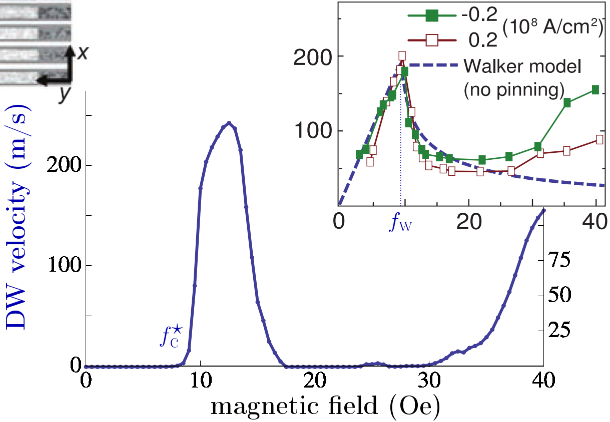
magnetic field (Oe)

Experiment (i)

SPINTRONICS

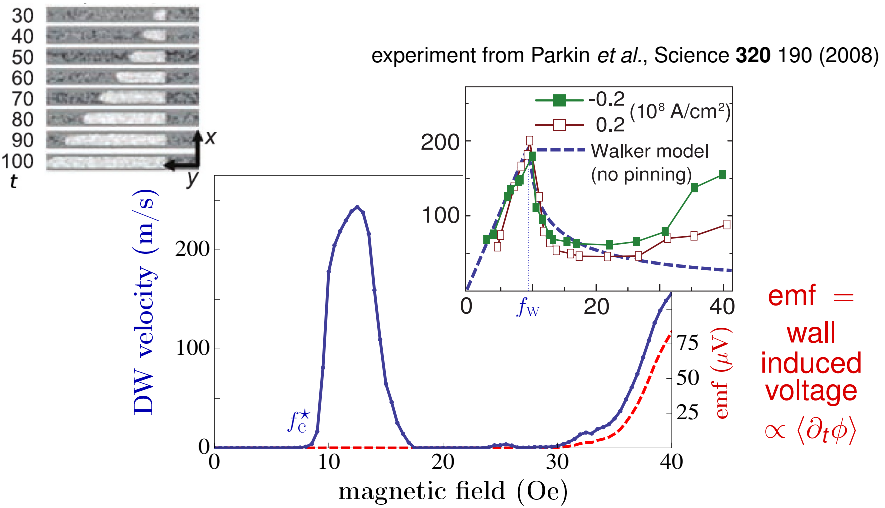


experiment from Parkin *et al.*, Science **320** 190 (2008)



Experiment (i)

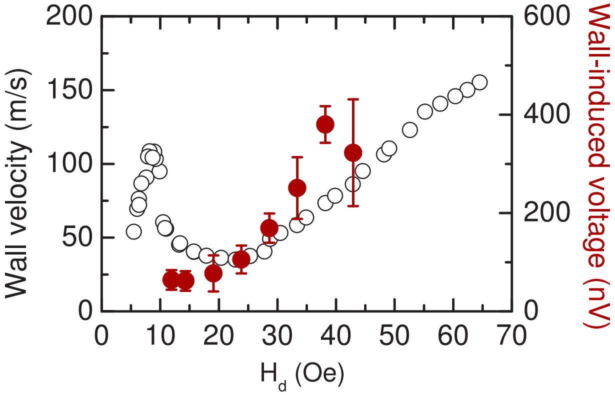
SPINTRONICS



emf = wall induced voltage $\propto \langle \partial_t \phi \rangle$

Experiment (ii) SPINTRONICS

experiment from Yang, Beach *et al.*, PRL **102** 067201 (2009)



$\propto \langle \partial_t \phi \rangle$

Including elasticity (full 1D interface)

[Ongoing work with Alejandro Kolton, Victor Purrello]

Interface with a **inertia** (velocity $=$ internal degree of freedom)

- Is there a critical mass m_c? (i.e. $m > m_c \Rightarrow$ depinning changes)
  $\longrightarrow$ What is the phase diagram?
- How is the depinning affected?
  $\longrightarrow$ Bistability? Finite depinning exponent β?

Including elasticity (full 1D interface)

[Ongoing work with Alejandro Kolton, Victor Purrello]

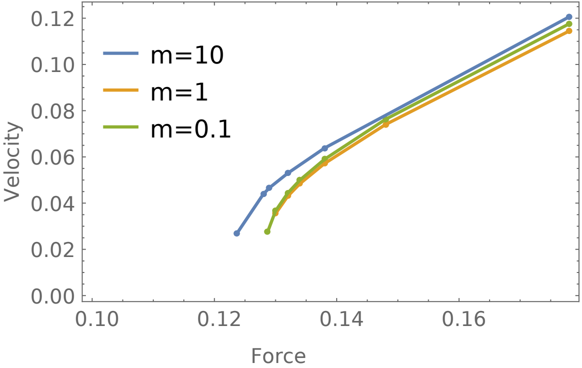
Interface with a **inertia** (velocity $=$ internal degree of freedom)

- Is there a critical mass m_c? (i.e. $m > m_c \Rightarrow$ depinning changes)
  $\longrightarrow$ What is the phase diagram?
- How is the depinning affected?
  $\longrightarrow$ Bistability? Finite depinning exponent β?
- How are geometrical properties affected?
- How, in presence of disorder, can there be a **cooperation** between position and velocity?
- What are the effects of temperature?

Including elasticity (full 1D interface)

[Ongoing work with Alejandro Kolton, Victor Purrello]

Interface with a **inertia** (velocity = internal degree of freedom)

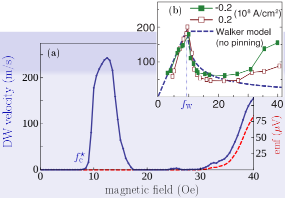


Summary

PRB **80** 054413 (2009)

Internal degree of freedom

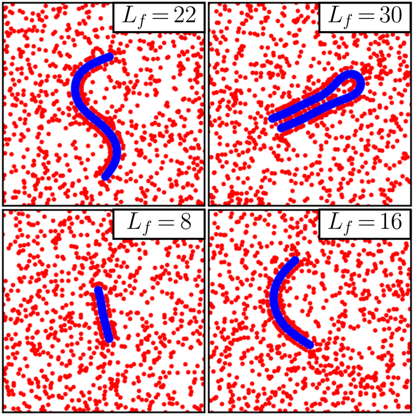
- unusual depinning law
- bistability at zero *T*
- non-monotonous $v(f)$ at finite *T*



Outline

Interfaces in active materials

[Nikolai Nikola, Alexandre P. Solon, Yariv Kafri, Mehran Kardar, Julien Tailleur, Raphaël Voituriez
PRL **117** 098001 (2016)]



Interfaces in active materials

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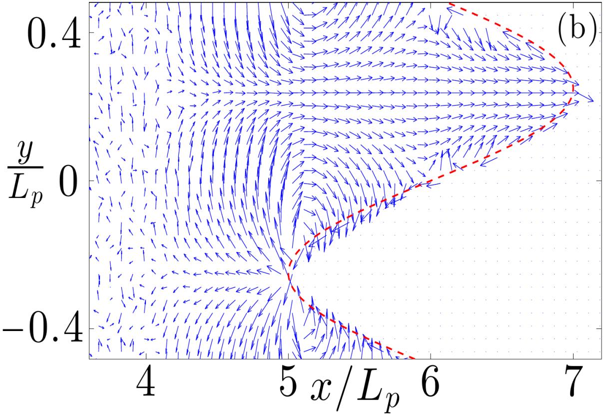

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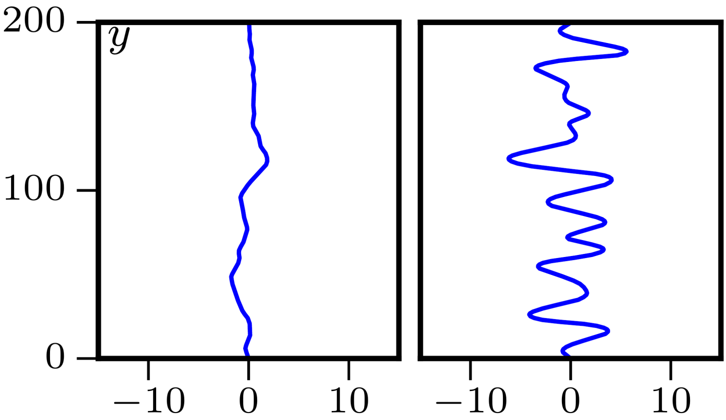


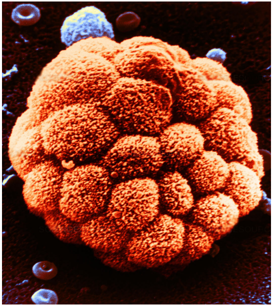
Interfaces in active materials

[Nikolai Nikola, Alexandre P. Solon, Yariv Kafri, Mehran Kardar, Julien Tailleur, Raphaël Voituriez
PRL **117** 098001 (2016)]

▶ **Issues**:
- Energy is injected and dissipated in the bulk.
- Usual concepts of thermodynamics (pressure,. . .) do not apply.

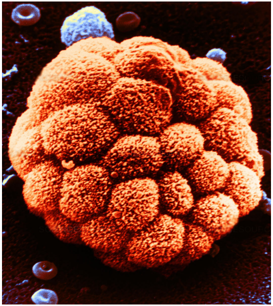
▶ **Questions**:
- How to understand fluctuations of interfaces/walls?
- How to build effective models?

Interfaces dynamics of tissues



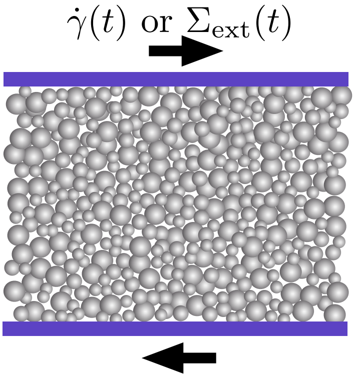
$$\xrightarrow{?}$$

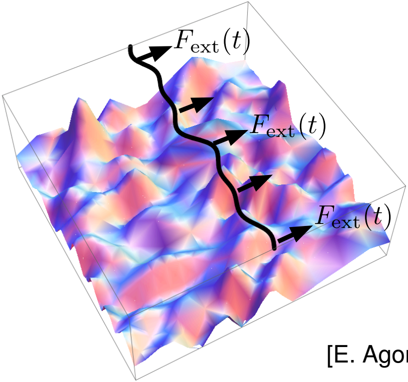
Interfaces dynamics of tissues



$\xrightarrow{?}$

Relation between yielding and depinning





[E. Agoritsas]

- Yielding transition $\leftrightarrow$ depinning transition
- Burst dynamics $\leftrightarrow$ avalanche dynamics

Thank you for your attention!

Additional slides

Equilibrium directed polymer at temperature T (and $f = 0$)

- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\text{el}}[y(t), t_{\text{f}}] = \frac{c}{2} \int_0^{t_{\text{f}}} dt \left[\partial_t y(t)\right]^2$$

- Disorder: tends to bend it

$$\mathcal{H}_V^{\text{dis}}[y(t), t_{\text{f}}] = \int_0^{t_{\text{f}}} dt \, V\big(t, y(t)\big)$$

Competition btw “order” and “disorder”

Equilibrium directed polymer at temperature T (and $f = 0$)

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- Disorder: tends to bend it

$$\mathcal{H}_V^{\mathrm{dis}}[y(t), t_{\mathrm{f}}] = \int_0^{t_{\mathrm{f}}} dt\, V\big(t, y(t)\big)$$

weight of $(y(t))_{0<t<t_{\mathrm{f}}}$:

$$e^{-\mathcal{H}_V/T}$$

with

$$\mathcal{H}_V = \mathcal{H}^{\mathrm{el}} + \mathcal{H}_V^{\mathrm{dis}}$$

Competition btw "order" and "disorder"

- Interpretations of the elastic Hamiltonian:

elasticity or kinetic energy or Wiener measure

Equilibrium directed polymer at temperature T (and $f = 0$)

- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\rm el}[y(t),t_{\rm f}] = \frac{c}{2}\int_0^{t_{\rm f}} dt\,\big[\partial_t y(t)\big]^2$$

- Disorder: tends to bend it

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weight of $(y(t))_{0<t<t_{\rm f}}$:

$$e^{-\mathcal{H}_V/T}$$

with

$$\mathcal{H}_V = \mathcal{H}^{\rm el} + \mathcal{H}_V^{\rm dis}$$

Competition btw “order” and “disorder”

- Ingredients up to now:

elastic constant c disorder potential $V(t,y)$ temperature T

A naive scaling argument 1/3

Roughness function (variance of the end-point fluctuations):

$$B(t_{\mathrm{f}}) = \overline{\langle y(t_{\mathrm{f}})^2 \rangle} \qquad \overline{\,\cdot\,} = \text{disorder average} \qquad \langle \cdot \rangle = \text{thermal average}$$

Path-integral writing:

$$B(t_{\mathrm{f}}) = \int \mathcal{D}V \; \mathbb{P}[V] \, \frac{\int_{y(0)=0} \mathcal{D}y(t)\, y(t_{\mathrm{f}})^2 \, \mathrm{e}^{-\frac{1}{T}\mathcal{H}[y(t),V;t_{\mathrm{f}}]}}{\int_{y(0)=0} \mathcal{D}y(t)\, \mathrm{e}^{-\frac{1}{T}\mathcal{H}[y(t),V;t_{\mathrm{f}}]}}$$

Uncorrelated disorder: Gaussian, centered, with

$$\overline{V(t,y)V(t',y')} = D\,\delta(t'-t)\delta(y'-y)$$

which rescales as

$$V(b\,\hat{t}, a\,\hat{y}) \stackrel{d}{=} a^{-\frac{1}{2}} b^{-\frac{1}{2}} D^{\frac{1}{2}}\, \hat{V}(\hat{t},\hat{y})$$

A naive scaling argument 2/3

Flory rescaling: $t = t_{\mathrm{f}}\,\hat{t}, \qquad y = t_{\mathrm{f}}^{\zeta_{\mathrm{F}}}\left(\frac{D}{c^2}\right)^{\frac{1}{5}}\hat{y}, \qquad \zeta_{\mathrm{F}} = \frac{3}{5}$
ensures

$$\frac{1}{T}\mathcal{H}^{\mathrm{el}} = \frac{(cD^2)^{1/5}}{T}t_{\mathrm{f}}^{1/5}\,\frac{1}{2}\int_0^1 d\hat{t}\,\left[\partial_{\hat{t}}\hat{y}(\hat{t})\right]^2$$

$$\frac{1}{T}\mathcal{H}^{\mathrm{dis}} \overset{d}{=} \frac{(cD^2)^{1/5}}{T}t_{\mathrm{f}}^{1/5}\int_0^1 d\hat{t}\;\hat{V}\big(\hat{t},\hat{y}(\hat{t})\big)$$

with correlations:

$$\overline{\hat{V}(\hat{t},\hat{y})\hat{V}(\hat{t}',\hat{y}')} = \delta(\hat{t}'-\hat{t})\delta(\hat{y}'-\hat{y})$$

A naive scaling argument 2/3

Flory rescaling: $t = t_{\mathrm{f}}\,\hat{t}, \qquad y = t_{\mathrm{f}}^{\zeta_{\mathrm{F}}} \big(\frac{D}{c^2}\big)^{\frac{1}{5}} \hat{y}, \qquad \zeta_{\mathrm{F}} = \frac{3}{5}$
Rescaling of the roughness

$$B(t_{\mathrm{f}}) = \left[\frac{D}{c^2}\right]^{2/5} t_{\mathrm{f}}^{6/5}\,\hat{B}(t_{\mathrm{f}})$$

$$\hat{B}(t_{\mathrm{f}}) = \frac{\displaystyle\int_{\hat{y}(0)=0} \mathcal{D}\hat{y}(\hat{t})\,\hat{y}(1)^2 \exp\Big\{-\frac{(cD^2)^{\frac{1}{5}}}{T} t_{\mathrm{f}}^{\frac{1}{5}} \int_0^1 d\hat{t}\,\Big[\tfrac{1}{2}(\partial_{\hat{t}}\hat{y})^2 + \hat{V}(\hat{t},\hat{y}(\hat{t}))\Big]\Big\}}{\displaystyle\int_{\hat{y}(0)=0} \mathcal{D}\hat{y}(\hat{t}) \exp\Big\{-\frac{(cD^2)^{\frac{1}{5}}}{T} t_{\mathrm{f}}^{\frac{1}{5}} \int_0^1 d\hat{t}\,\Big[\tfrac{1}{2}(\partial_{\hat{t}}\hat{y})^2 + \hat{V}(\hat{t},\hat{y}(\hat{t}))\Big]\Big\}}$$

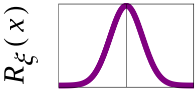
If saddle-point trajectory $\hat{y}^{\star}(\hat{t})$ exists at $t_{\mathrm{f}} \to \infty$, **it is indep. of t_{f}**

$$B(t_{\mathrm{f}}) = \left[\frac{D}{c^2}\right]^{2/5} t_{\mathrm{f}}^{6/5}\,\overline{\hat{y}^{\star}(1)^2} \quad \text{not the expected KPZ behaviour} \quad \sim t_{\mathrm{f}}^{4/3}$$

A naive scaling argument 3/3

Where is t_f? In the disorder correlations on a lengthscale ξ:

$$\overline{V(z,x)V(z',x')} = D\,\delta(z'-z)R_\xi(x'-x)$$



scaling as $R_\xi(x) = \frac{1}{\xi}\, R_{\xi=1}(x/\xi)$

A naive scaling argument 3/3

Where is t_{f}? In the disorder correlations on a lengthscale ξ:

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Rescaling of the roughness

$$B(t_{\mathrm{f}}) = \left[\tfrac{D}{c^2}\right]^{2/5} t_{\mathrm{f}}^{6/5}\,\hat{B}(t_{\mathrm{f}}) \qquad \text{where denoting} \quad \hat{\xi}(t_{\mathrm{f}}) = \frac{\xi}{t_{\mathrm{f}}^{\zeta_{\mathrm{F}}}\left(\frac{D}{c^2}\right)^{\frac{1}{5}}}$$

$$\hat{B}(t_{\mathrm{f}}) = \overline{\frac{\int_{\hat{y}(0)=0}\mathcal{D}\hat{y}(\hat{t})\,\hat{y}(1)^2 \exp\left\{-\frac{(cD^2)^{\frac{1}{5}}}{T}t_{\mathrm{f}}^{\frac{1}{5}}\int_0^1 d\hat{t}\left[\frac{1}{2}(\partial_{\hat{t}}\hat{y})^2 + \hat{V}_{\hat{\xi}(t_{\mathrm{f}})}(\hat{t},\hat{y}(\hat{t}))\right]\right\}}{\int_{\hat{y}(0)=0}\mathcal{D}\hat{y}(\hat{t})\exp\left\{-\frac{(cD^2)^{\frac{1}{5}}}{T}t_{\mathrm{f}}^{\frac{1}{5}}\int_0^1 d\hat{t}\left[\frac{1}{2}(\partial_{\hat{t}}\hat{y})^2 + \hat{V}_{\hat{\xi}(t_{\mathrm{f}})}(\hat{t},\hat{y}(\hat{t}))\right]\right\}}}$$

$$B(t_{\mathrm{f}}) = \left[\tfrac{D}{c^2}\right]^{2/5} t_{\mathrm{f}}^{6/5}\,\overline{\hat{y}^\star(1)^2}$$

{ the $t_{\mathrm{f}} \to \infty$ saddle-point trajectory $\hat{y}^\star(\hat{t})$ **depends on t_{f} through $\hat{\xi}(t_{\mathrm{f}})$** }

A naive scaling argument 3/3

Where is $t_{\rm f}$? In the disorder correlations on a lengthscale ξ:

$$\overline{V(z,x)V(z',x')} = D\,\delta(z'-z)R_\xi(x'-x)$$

Flory rescaling: $t = t_{\rm f}\,\hat{t}, \qquad y = t_{\rm f}^{\zeta_{\rm F}}\left(\frac{D}{c^2}\right)^{\frac15}\hat{y}, \qquad \zeta_{\rm F} = \frac35$

Rescaling of the roughness

$$B(t_{\rm f}) = \left[\tfrac{D}{c^2}\right]^{2/5} t_{\rm f}^{6/5}\,\hat{B}(t_{\rm f}) \qquad \text{where denoting} \quad \hat{\xi}(t_{\rm f}) = \frac{\xi}{t_{\rm f}^{\zeta_{\rm F}}\left(\frac{D}{c^2}\right)^{\frac15}}$$

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$$B(t_{\rm f}) = \left[\tfrac{D}{c^2}\right]^{2/5} t_{\rm f}^{6/5}\,\overline{\hat{y}^\star(1)^2} \qquad \Big\{\cdots\Big\} \Rightarrow \text{modifies the exponent } \frac65 \mapsto \frac43$$

Free-energy fluctuations of trajectories $(0,0) \rightsquigarrow (t,y)$

How to solve this issue?

- Partition function Z_V *vs.* Free-energy F_V

$$Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t')\mathrm{e}^{-\frac{1}{T}\mathcal{H}_V[y(t'),t]} \qquad F_V(t,y) = -\frac{1}{T}\log Z_V(t,y)$$

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- **Stochastic Heat Equation** (Feynman-Kac formula)

$$\partial_t Z_V = \Big[\frac{T}{2c}\partial_y^2 - \frac{1}{T}V(t,y)\Big] Z_V(t,y) \qquad \textbf{(SHE)}$$

Linear, multiplicative noise, reversible

- **Kardar-Parisi-Zhang** equation

$$\partial_t F_V = \frac{T}{2c}\partial_y^2 F_V - \frac{1}{2c}\big[\partial_y F_V\big]^2 + V(t,y) \qquad \textbf{(KPZ)}$$

Non-linear, additive noise, non-reversible

$F_V(t,y) \equiv$ interface height at position y and time t

Free-energy fluctuations of trajectories $(0,0) \rightsquigarrow (t,y)$

How to solve this issue?

- Partition function Z_V *vs.* Free-energy F_V

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Linear, multiplicative noise, $Z_V(0,y) = \delta(y)$

- **Kardar-Parisi-Zhang** equation

$$\partial_t F_V = \frac{T}{2c}\partial_y^2 F_V - \frac{1}{2c}\big[\partial_y F_V\big]^2 + V(t,y) \qquad \textbf{(KPZ)}$$

Non-linear, additive noise, $F_V(0,y)$: "sharp wedge" initial cond.
$F_V(t,y) \equiv$ interface height at position y and time t

Statistical tilt symmetry

How to solve this issue?

- Partition function Z_V *vs.* Free-energy F_V

$$Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t') \mathrm{e}^{-\frac{1}{T}\mathcal{H}_V[y(t'),t]} \qquad F_V(t,y) = -\frac{1}{T}\log Z_V(t,y)$$

Statistical tilt symmetry

How to solve this issue?

- Partition function Z_V *vs.* Free-energy F_V

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- **Statistical Tilt Symmetry**

$$F_V(t,y) = \underbrace{c\frac{y^2}{2t} + \frac{T}{2}\log\frac{2\pi Tt}{c}}_{\substack{\text{thermal contribution} \\ F_{V\equiv 0}}} + \underbrace{\bar{F}_V(t,y)}_{\substack{\text{disorder} \\ \text{contribution}}} \qquad \textbf{(STS)}$$

- **Tilted** KPZ equation for $\bar{F}_V(t,y)$

$$\partial_t \bar{F}_V + \frac{y}{t}\partial_y \bar{F}_V = \frac{T}{2c}\partial_y^2 \bar{F}_V - \frac{1}{2c}\left[\partial_y \bar{F}_V\right]^2 + V(t,y)$$

Non-linear, additive noise, $\bar{F}_V(0,y) \equiv 0$: “simple” initial cond.

Known results $@\xi = 0$ $[\Longleftrightarrow T \to \infty \; @\xi > 0]$

- **Infinite-time limit** $t_{\mathrm{f}} \to \infty$ (steady state)

  $\bar{F}(t_{\mathrm{f}} = \infty, y)$ distributed as a 2-sided Brownian Motion

  i.e.: $\mathbb{P}\big[\bar{F}(t_{\mathrm{f}} = \infty, y)\big]$ Gaussian, of correlator

$$\overline{\big[\bar{F}(t_{\mathrm{f}} = \infty, y) - \bar{F}(t_{\mathrm{f}} = \infty, y')\big]^2} = \widetilde{D}\,|y - y'| \quad \text{with} \quad \boxed{\widetilde{D} = \frac{cD}{T}}$$

Known results $@\xi = 0$ $[\Longleftrightarrow T \to \infty\ @\xi > 0]$

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- Rescaling of the disorder free-energy $\bar{F}(t_{\mathrm{f}} = \infty, a\hat{y})$

$$\bar{F}(t_{\mathrm{f}} = \infty, a\hat{y}) \stackrel{d}{=} a^{1/2}\widetilde{D}^{1/2} \underbrace{\hat{F}(t_{\mathrm{f}} = \infty, \hat{y})}_{\widetilde{D}=1}$$

Saddle-point argument for the KPZ exponent

Flory rescaling for the free-energy

$$t = t_{\mathrm{f}}\hat{t}, \qquad y = (\widetilde{D}/c^2)^{\frac{1}{3}}\, t_{\mathrm{f}}^{\frac{2}{3}}\, \hat{y}, \qquad \bar{F}_V(t,y) \overset{(d)}{=} (\widetilde{D}^2 t_{\mathrm{f}}/c)^{1/3}\hat{F}(\hat{t},\hat{y})$$

Rescaling of the roughness

$$B(t_{\mathrm{f}}) \underset{t_{\mathrm{f}}\to\infty}{\sim} \left[\frac{\widetilde{D}}{c^2}\right]^{\frac{2}{3}} t_{\mathrm{f}}^{\frac{4}{3}}\ \hat{B}(t_{\mathrm{f}})$$

$$\hat{B}(t_{\mathrm{f}}) = \overline{\frac{\int_{\mathbb{R}} d\hat{y}\ \hat{y}^2 \exp\left\{-\frac{1}{T}\left(\frac{\widetilde{D}^2}{c}t_{\mathrm{f}}\right)^{\frac{1}{3}}\left[\frac{\hat{y}^2}{2}+\hat{F}(\hat{t},\hat{y})\right]\right\}}{\int_{\mathbb{R}} d\hat{y}\ \exp\left\{-\frac{1}{T}\left(\frac{\widetilde{D}^2}{c}t_{\mathrm{f}}\right)^{\frac{1}{3}}\left[\frac{\hat{y}^2}{2}+\hat{F}(\hat{t},\hat{y})\right]\right\}}}$$

The $t_{\mathrm{f}} \to \infty$ saddle-point [$\hat{B}(t_{\mathrm{f}}) \sim \overline{(\hat{y}^{\star})^2} \sim t_{\mathrm{f}}^0$] gives the correct exponent