Large deviations functions and dynamical phase transitions

Vivien Lecomte

LPMA, Paris

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Outline

1. Motivations
2. Kinetically Constrained Models

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3. Transport models

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Alberto Imparato[4], Frédéric van Wijland[5]
Julien Tailleur[6], Jorge Kurchan[7]

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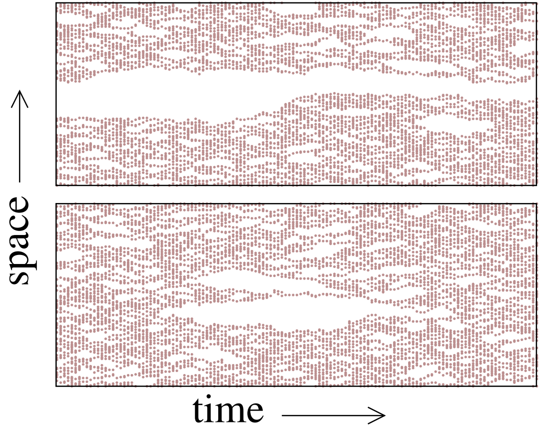
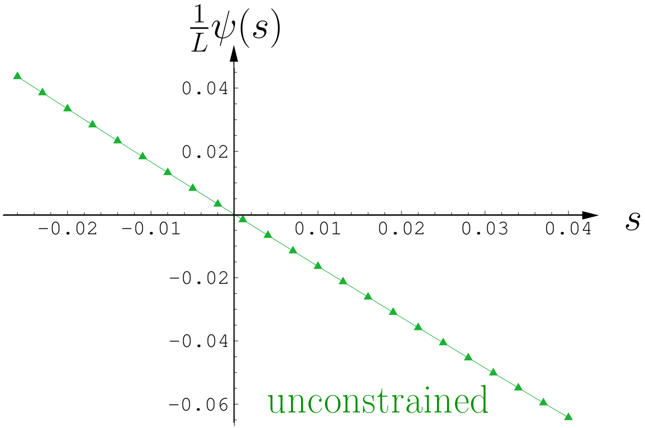
[5]MSC, Paris [6]School of Physics, Edinburgh [7]ESPCI, Paris

Space-time "bubbles" of inactivity



From: Merolle, Garrahan and Chandler, PNAS **102**, 10837 (2005)

Dynamical phase transition: FA model (d=1)



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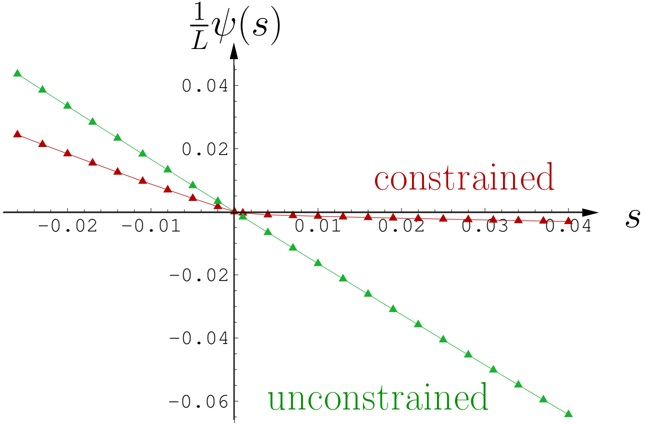

Comparison between constrained and unconstrained dynamics

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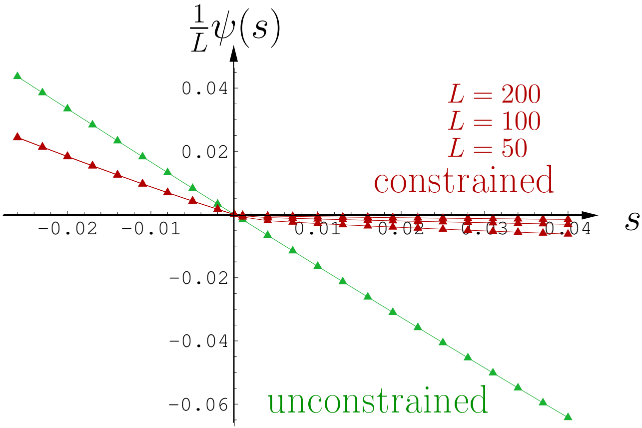
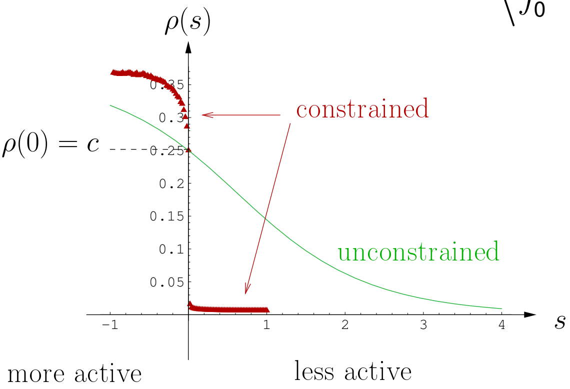


Comparison between constrained and unconstrained dynamics

Dynamical phase transition: FA model (d=1)

$$\rho(s) = \left\langle \int_0^t d\tau \frac{1}{L} \sum_i n_i \right\rangle_s$$



Comparison between constrained and unconstrained dynamics

Dynamical Landau free energy $\mathcal{F}(\rho, s)$

Probability, in the *s*-state, to measure a time-averaged density ρ btw. 0 and *t* $\sim e^{-t\,L\,\mathcal{F}(\rho,s)}$

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$$\left\langle e^{-sK}\delta\big(\langle n\rangle - L\rho\big)\right\rangle\Big| \sim e^{-t\,L\,\mathcal{F}(\rho,s)}$$

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Extremalization procedure

$$\psi(s) = -\min_{\rho}\mathcal{F}(\rho, s)$$

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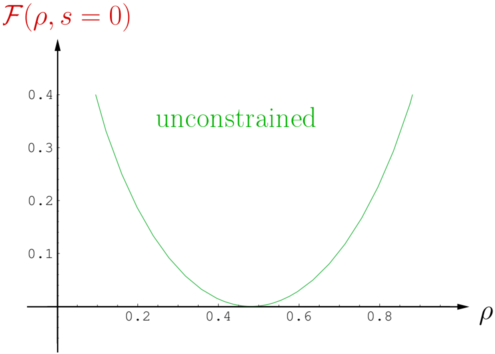
Minimum reached at $\rho = \rho(s)$:

$$\psi(s) = \mathcal{F}(\rho(s), s)$$

Dynamical Landau free-energy landscape



$$\mathrm{Prob}_{[0,t]}(\rho) \sim \mathrm{e}^{-t\,L\,\mathcal{F}(\rho,s)}$$

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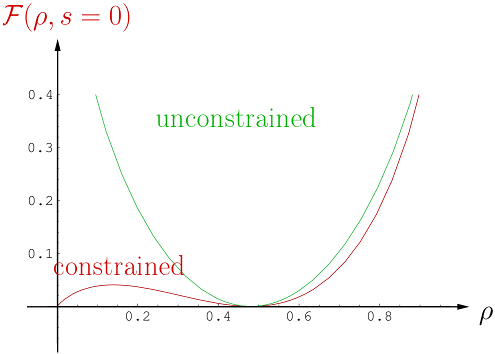

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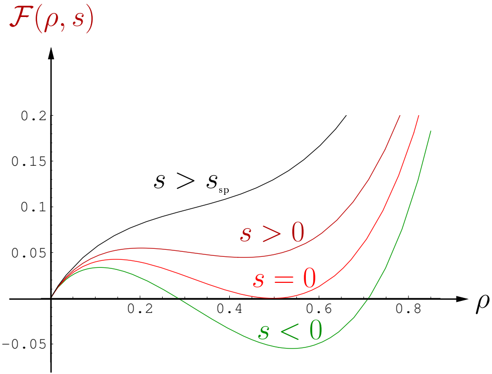
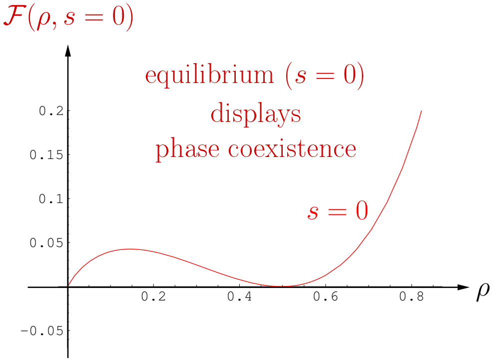


$$\psi(s) = -\min_{\rho} \mathcal{F}(\rho, s) = -\mathcal{F}(\rho(s), s)$$

Dynamical Landau free-energy landscape

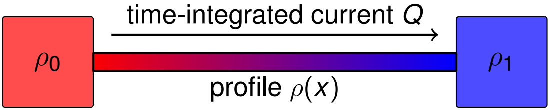


$$\text{Prob}_{[0,t]}(\rho) \sim e^{-t\,L\,\mathcal{F}(\rho,s)}$$

Open questions

- Finite-size scaling
- Results in finite dimension
- Measurable ($s = 0$) consequences of the transition, at finite time

Questions



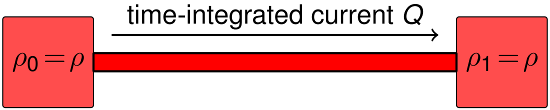
- Non-equilibrium steady-state

$$\text{Prob}[\rho(x)]$$

- (Non-)equilibrium fluctuations of dynamical observables

$$\text{Prob}[Q]$$

Questions



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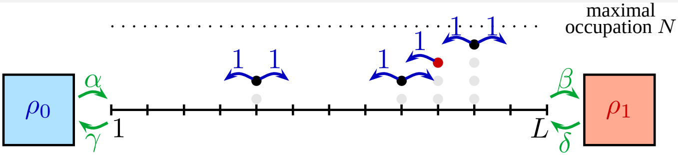
Exclusion Processes



- Configurations: occupation numbers $\{n_i\}$
- Exclusion rule: $0 \leq n_i \leq N$
- Markov evolution

$$\partial_t P(\{n_i\}) = \sum_{n_i'} \left[W(n_i' \to n_i) P(\{n_i'\}) - W(n_i \to n_i') P(\{n_i\}) \right]$$

- Large deviation function of the time-integrated current Q

$$\langle e^{-sQ} \rangle \sim e^{t\psi(s)} \qquad (\Leftrightarrow \text{determining } P(Q))$$

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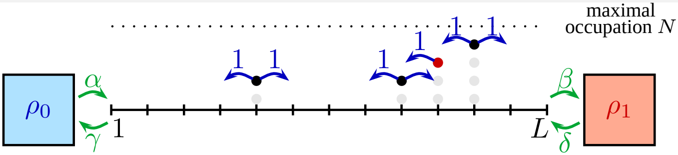

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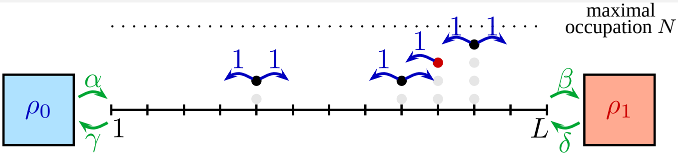
Operator representation

[Schütz & Sandow PRE **49** 2726]



$$\partial_t P \; = \; \mathbb{W}\, P$$

$$\mathbb{W} = \sum_{1\le k\le L-1} \left[S_k^+ S_{k+1}^- + S_k^- S_{k+1}^+ - \hat{n}_k(1-\hat{n}_{k+1}) - \hat{n}_{k+1}(1-\hat{n}_k) \right] + \alpha\left[S_1^+ - (1-\hat{n}_1)\right] + \gamma\left[S_1^- - \hat{n}_1\right] + \delta\left[S_L^+ - (1-\hat{n}_L)\right] + \beta\left[S_L^- - \hat{n}_L\right]$$

$S^\pm$ and **creation and annihilation operators**:

$$S^+|n\rangle = (N-n)|n+1\rangle \qquad S^-|n\rangle = n|n-1\rangle$$

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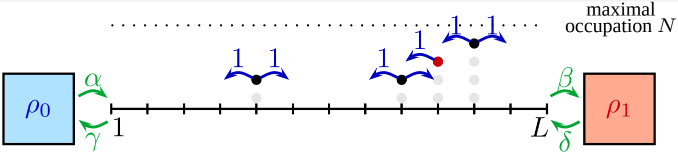

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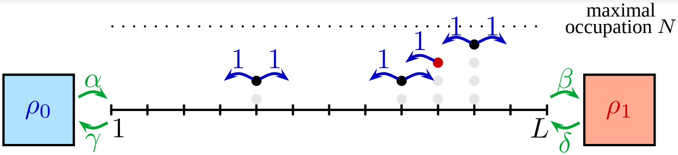
$S^\pm$ and $S^z = \hat{n} - \frac{N}{2}$ are **spin operators** (with $j = \frac{N}{2}$)

Operator representation



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Bethe Ansatz method [Appert, Derrida, VL, van Wijland, PRE **78** 021122]

SSEP: maximal occupation $N = 1$
Periodic boundary conditions

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Bethe Ansatz:

- eigenvector of components

$$\sum_{\mathcal{P}} \mathcal{A}(\mathcal{P}) \prod_{i=1}^{\mathcal{N}} \left[\zeta_{\mathcal{P}(i)}\right]^{x_i}$$

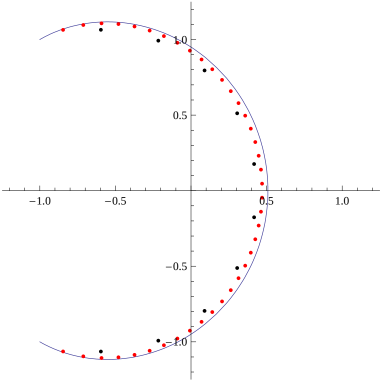
- eigenvalue

$$\psi(s) = -2\mathcal{N} + e^{-s}\left[\zeta_1 + \ldots + \zeta_{\mathcal{N}}\right] - e^{s}\left[\frac{1}{\zeta_1} + \ldots + \frac{1}{\zeta_{\mathcal{N}}}\right]$$

- Bethe equations

$$\zeta_i^L = \prod_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \left[-\frac{1 - 2e^{-s}\zeta_i + e^{-2s}\zeta_i\zeta_j}{1 - 2e^{-s}\zeta_j + e^{-2s}\zeta_i\zeta_j}\right]$$

Bethe Ansatz method [Appert, Derrida, VL, van Wijland, PRE **78** 021122]



Repartition of Bethe roots in the complex plane

Finite-size effects

[Appert, Derrida, VL, van Wijland, PRE **78** 021122]

- large deviation function

$$\psi(s) = \underbrace{\frac{1}{L}\rho(1-\rho)s^2}_{\text{order 0}} + \underbrace{L^{-2}\mathcal{F}(u)}_{\text{finite-size}} \quad \text{with} \quad u = -\tfrac{1}{2}\rho(1-\rho)s^2$$

- universal function

$$\mathcal{F}(u) = \sum_{k\geq 2} \frac{(-2u)^k B_{2k-2}}{\Gamma(k)\Gamma(k+1)}$$

- scaling of cumulants for the total current

$$\tfrac{1}{t}\langle Q^2\rangle \sim L$$

$$\tfrac{1}{t}\langle Q^{2k}\rangle \sim L^{2k-2} \qquad (k \geq 2)$$

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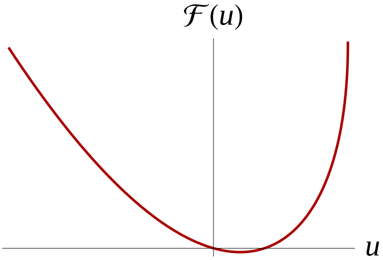
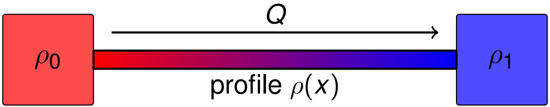


Macroscopic limit

[Tailleur, Kurchan, VL, JPA **41** 505001]



Using $SU(2)$ coherent states:

$$\langle \rho_\mathrm{f} | e^{t\mathbb{W}} | \rho_\mathrm{i} \rangle = \int_{\rho(0)=\rho_\mathrm{i}}^{\rho(t)=\rho_\mathrm{f}} \mathcal{D}\rho \mathcal{D}\hat{\rho}\ \exp\{ L\, \underbrace{\mathcal{S}[\hat{\rho},\rho]}_{\text{action}} \}$$

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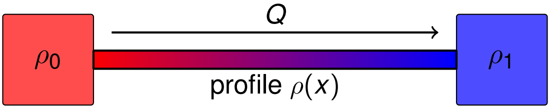

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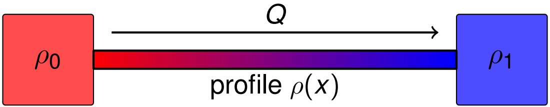

Same $\mathcal{S}_s[\hat{\rho}, \rho]$ as the MSR action of the Langevin evolution:

$$\partial_t \rho = -\partial_x \big[-\partial_x \rho + \xi \big]$$

$$\langle \xi(x,t)\xi(x',t')\rangle = \underbrace{\frac{1}{L}\rho(1-\rho)}_{\text{density-dependent}} \quad \delta(x'-x)\delta(t'-t)$$

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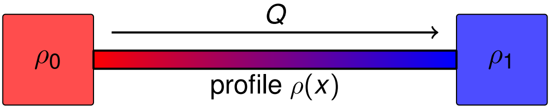

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One recovers the action of fluctuating hydrodynamics

[Bertini De Sole Gabrielli Jona-Lasinio Landim]

$\psi(s)$: again [Appert, Derrida, VL, van Wijland, PRE **78** 021122]

Periodic boundary conditions
More general fluctuating hydrodynamics

$$\frac{1}{Lt}\langle Q\rangle \propto D(\rho) \qquad \text{(Fourier's law)}$$

$$\frac{1}{Lt}\langle Q^2\rangle_c = \sigma(\rho) \qquad \text{(For the SSEP, } \sigma = \rho(1-\rho)\text{)}$$

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Saddle point evaluation

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Large deviation function

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Correspondence between
(Gaussian) integration of small fluctuations
AND
discreteness of Bethe root repartition.

More general?

With a field [Appert, Derrida, VL, van Wijland, PRE **78** 021122]

Periodic boundary conditions
Driving field E

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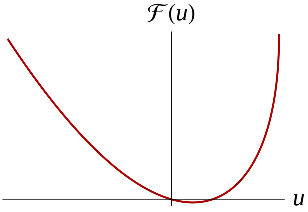
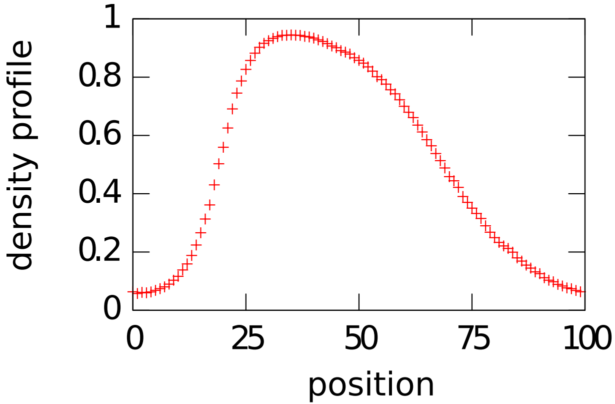


Dynamical phase transition
between
stationnary and non-stationnary
profiles

After the transition (numerical approach)



Non-steady and non-unifor density profile

Microscopic approach [Imparato, VL, van Wijland, PTP **184** 276 (2010)]

Large deviations of the current $\psi(s) = \max \mathrm{Sp}\, \mathbb{W}(s)$

$$\mathbb{W}(s) = \overbrace{\sum_{1\le k\le L-1} \vec{S}_k \cdot \vec{S}_{k+1}}^{\text{invariant by rotation}} + \alpha\left[S_1^+ - (1-\hat{n}_1)\right] + \gamma\left[S_1^- - \hat{n}_1\right] + \delta\left[S_L^+ e^{s} - (1-\hat{n}_L)\right] + \beta\left[S_L^- e^{-s} - \hat{n}_L\right]$$

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Local transformation

$$\mathcal{Q}^{-1} \mathbb{W}(s) \mathcal{Q} = \sum_{1 \le k \le L-1} \vec{S}_k \cdot \vec{S}_{k+1}$$
$$+ \alpha' \left[S_1^+ - (1 - \hat{n}_1) \right] + \gamma' \left[S_1^- - \hat{n}_1 \right]$$
$$\underbrace{+ \delta' \left[S_L^+ e^{s'} - (1 - \hat{n}_L) \right] + \beta' \left[S_L^- e^{-s'} - \hat{n}_L \right]}_{\text{describes contact with reservoirs of same densities}}$$

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Local transformation Fluctuations@eq $\leftrightarrow$ Fluctuations@non-eq

$$\mathcal{Q}^{-1}\mathbb{W}(s)\mathcal{Q} = \sum_{1\le k\le L-1} \vec{S}_k \cdot \vec{S}_{k+1} + \underbrace{\alpha'\left[S_1^+ - (1-\hat{n}_1)\right] + \gamma'\left[S_1^- - \hat{n}_1\right] + \delta'\left[S_L^+ e^{s'} - (1-\hat{n}_L)\right] + \beta'\left[S_L^- e^{-s'} - \hat{n}_L\right]}_{\text{describes contact with reservoirs of same densities}}$$

Macroscopic approach [Imparato, VL, van Wijland, **PRE** 80 011131]

$$\langle e^{-sQ}\rangle \sim \int \mathcal{D}\rho\mathcal{D}\hat{\rho}\ \exp\{L\,\mathcal{S}_s[\hat{\rho},\rho]\}$$

Fluctuations ϕ, $\hat{\phi}$ around the saddle

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Large deviation function

$$\psi(s) = \underbrace{\frac{1}{L}\mu(s)}_{\text{saddle}} + \overbrace{\underbrace{\frac{D}{8L^2}\mathcal{F}\left(\frac{\sigma''}{2D^2}\mu(s)\right)}_{\text{fluctuations}}}^{\text{same } \mathcal{F} \text{ as at eq.}}$$

Macroscopic approach [Imparato, VL, van Wijland, **PRE** 80 011131]

Physical remark

In contact with reservoirs, neither SSEP nor KMP present a 'dynamical phase transition' ($\Leftarrow$ no singularity in $\psi(s)$).

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- solve the *spatially-discrete* stationary saddle-point equations

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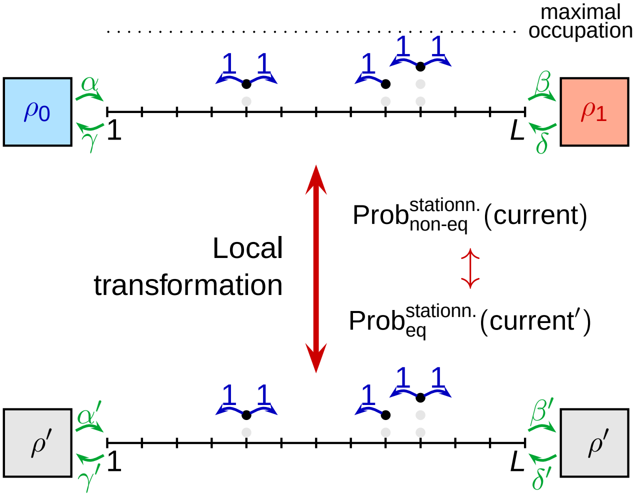
$\longrightarrow$ How does this translate in the Bethe Ansatz approach?

For the current

[Imparato, VL, van Wijland, **PRE** 80 011131]

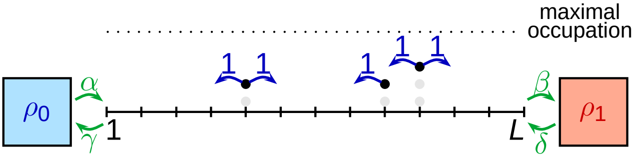
Symmetric exclusion process



System in equilibrium

For the density profile [Tailleur, Kurchan, VL, JPA **41** 505001]

Symmetric exclusion process

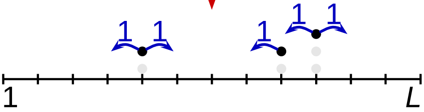


Non-local transformation

$\text{Prob}^{\text{stationn.}}_{\text{non-eq}}(\text{profile})$

$\updownarrow$

$\text{Prob}^{\text{stationn.}}_{\text{eq}}(\text{profile}')$



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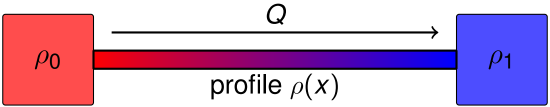
Non-local mapping

[Tailleur, Kurchan, VL, JPA **41** 505001]

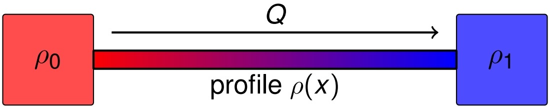


Boundary-driven transport model:

- long-range correlations
- breaking of time-reversal symmetry

Non-local mapping

[Tailleur, Kurchan, VL, JPA **41** 505001]



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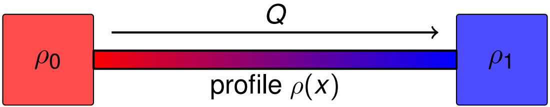
[Tailleur, Kurchan, VL, JPA **41** 505001]



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$\longrightarrow$ Applies to non-equilibrium quantum chains?

Summary

Approach:

- operator formalism
- large deviation function

Extensions:

- fluctuating hydrodynamics
- saddle-point method, instantons
- integration of fluctuations (dynamical phase transition)

Open questions:

$\rightarrow$ Eq$\leftrightarrow$non-eq mapping in higher dimensions?
$\rightarrow$ More generic systems of interacting particle?
$\rightarrow$ Link btw the eq$\leftrightarrow$non-eq mappings?
$\rightarrow$ Crossover to KPZ? Other universal fluctuations?

Other interests

Random manifolds/directed polymers:

- With E. Agoritsas & T. Giamarchi:
  roughness of interfaces in short-range correlated disorder
  functional renormalization group approach
- With J.-P. Eckmann & T. Giamarchi:
  depinning of interfaces with internal degrees of freedom

Fluctuation theorems:

- With R. Garcia-Garcia, A. Kolton, D. Dominguez:
  generic symmetries
  non-equilibrium fluctuation-dissipation relation