Large deviations in (non-)equilibrium 1d transport models

Cécile Appert-Rolland[1], Thierry Bodineau[2], Bernard Derrida[3], Alberto Imparato[4], Vivien Lecomte[5], Frédéric van Wijland[6]

Julien Tailleur[7], Jorge Kurchan[8]

[1]LPT, Orsay [2]DMA, Paris [3]LPS, Paris [4]DPA, Aarhus [5]LPMA, Paris & DPMC, Genève
[6]MSC, Paris [7]School of Physics, Edinburgh [8]ESPCI, Paris

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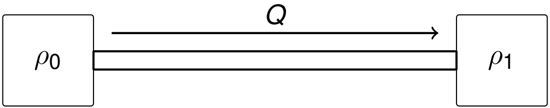
Outline

1. Motivations
2. Microscopic approach
   - Operator approach
   - Bethe Ansatz
3. Macroscopic approach
   - Fluctuating hydrodynamics
   - Finite-size corrections
4. Mapping non-equilibrium to equilibrium
   - For the integrated current
   - For the density profile

Motivations



$$\text{Prob}(\mathcal{C}) \propto \exp\left\{-\frac{\text{energy}(\mathcal{C})}{\text{temperature}}\right\}$$ cannot describe

- Non-equilibrium steady-state

$$\text{Prob}[\rho(x)]$$

- Equilibrium fluctuations of dynamical observables

$$\text{Prob}[Q]$$

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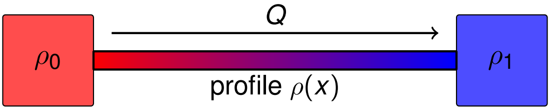

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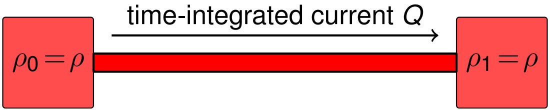

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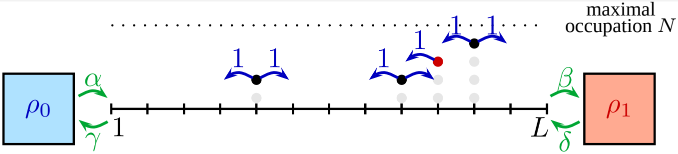
Exclusion Processes



- Configurations: occupation numbers $\{n_i\}$
- Exclusion rule: $0 \leq n_i \leq N$
- Markov evolution

$$\partial_t P(\{n_i\}) = \sum_{n'_i} \left[W(n'_i \to n_i) P(\{n'_i\}) - W(n_i \to n'_i) P(\{n_i\}) \right]$$

- Large deviation function of the time-integrated current Q

$$\langle e^{-sQ} \rangle \sim e^{t\psi(s)} \qquad (\Leftrightarrow \text{determining } P(Q))$$

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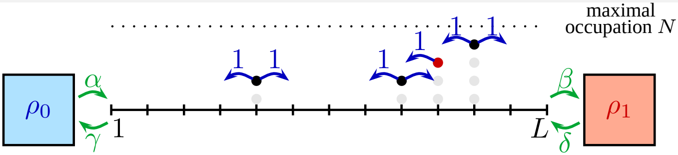

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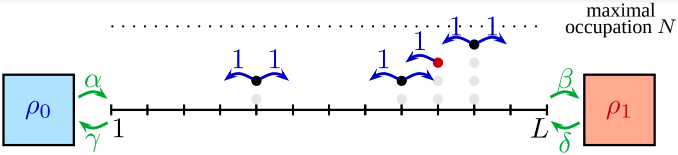

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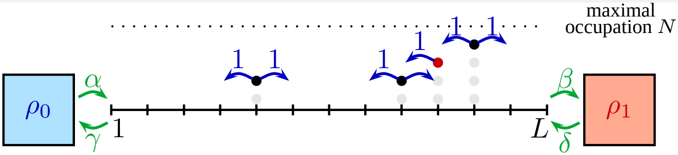
Operator representation

[Schütz & Sandow PRE **49** 2726]



$$\partial_t P = \mathbb{W} P$$

$$\mathbb{W} = \sum_{1\le k\le L-1} \left[S_k^+ S_{k+1}^- + S_k^- S_{k+1}^+ - \hat{n}_k(1-\hat{n}_{k+1}) - \hat{n}_{k+1}(1-\hat{n}_k)\right]$$
$$+ \alpha\left[S_1^+ - (1-\hat{n}_1)\right] + \gamma\left[S_1^- - \hat{n}_1\right]$$
$$+ \delta\left[S_L^+ - (1-\hat{n}_L)\right] + \beta\left[S_L^- - \hat{n}_L\right]$$

$S^\pm$ and **creation and annihilation operators**:

$$S^+|n\rangle = (N-n)|n+1\rangle \qquad S^-|n\rangle = n|n-1\rangle$$

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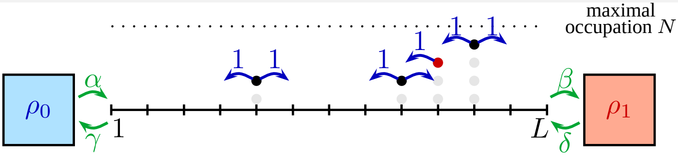

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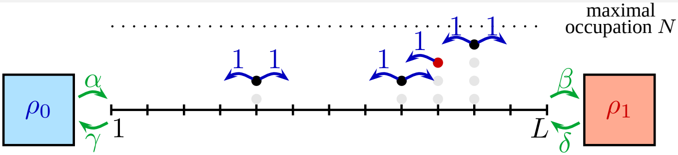
$$S^+|n\rangle = (N-n)|n+1\rangle \qquad S^-|n\rangle = n|n-1\rangle$$

$S^\pm$ and $S^z = \hat{n} - \frac{N}{2}$ are **spin operators** (with $j = \frac{N}{2}$)

Operator representation



$$\left\langle e^{-sQ}\right\rangle \sim e^{t\psi(s)} \qquad \text{with} \qquad \psi(s) = \max \mathrm{Sp}\, \mathbb{W}(s)$$

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Bethe Ansatz method [Appert, Derrida, VL, van Wijland, PRE **78** 021122]

SSEP: maximal occupation $N = 1$
Periodic boundary conditions

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Bethe Ansatz:

- eigenvector of components

$$\sum_{\mathcal{P}} \mathcal{A}(\mathcal{P}) \prod_{i=1}^{\mathcal{N}} \left[\zeta_{\mathcal{P}(i)}\right]^{x_i}$$

- eigenvalue

$$\psi(s) = -2\mathcal{N} + e^{-s}\left[\zeta_1 + \ldots + \zeta_{\mathcal{N}}\right] - e^{s}\left[\frac{1}{\zeta_1} + \ldots + \frac{1}{\zeta_{\mathcal{N}}}\right]$$

- Bethe equations

$$\zeta_i^L = \prod_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \left[-\frac{1 - 2e^{-s}\zeta_i + e^{-2s}\zeta_i\zeta_j}{1 - 2e^{-s}\zeta_j + e^{-2s}\zeta_i\zeta_j} \right]$$

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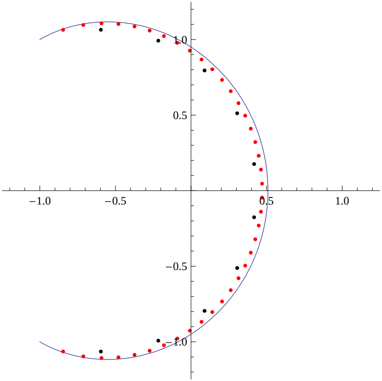

Repartition of Bethe roots in the complex plane

Finite-size effects

[Appert, Derrida, VL, van Wijland, PRE **78** 021122]

- large deviation function

$$\psi(s) = \underbrace{\frac{1}{L}\rho(1-\rho)s^2}_{\text{order 0}} + \underbrace{L^{-2}\mathcal{F}(u)}_{\text{finite-size}} \quad \text{with} \quad u = -\tfrac{1}{2}\rho(1-\rho)s^2$$

- universal function

$$\mathcal{F}(u) = \sum_{k\geq 2} \frac{(-2u)^k B_{2k-2}}{\Gamma(k)\Gamma(k+1)}$$

- scaling of cumulants for the total current

$$\tfrac{1}{t}\langle Q^2\rangle \sim L$$

$$\tfrac{1}{t}\langle Q^{2k}\rangle \sim L^{2k-2} \qquad (k \geq 2)$$

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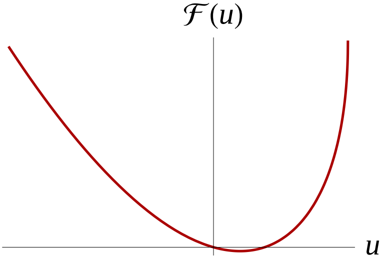

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- universal function
- technical remark:

  . in that context (*symmetric* jump rates), the results of Kim [PRE **52** 3512] do not apply

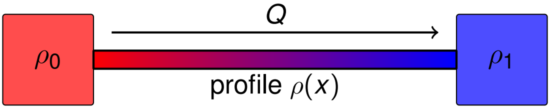
  . in particular the large L asymptotics reads

$$\frac{1}{L}\psi(Ls) = \frac{1}{2}\sigma\, s^2 + \sigma^{3/2}|s|^3$$

Outline

Macroscopic limit

[Tailleur, Kurchan, VL, JPA **41** 505001]



A reminder: propagator in quantum mechanics

$\langle \text{final} | e^{it\mathbb{H}} | \text{initial} \rangle$

Macroscopic limit

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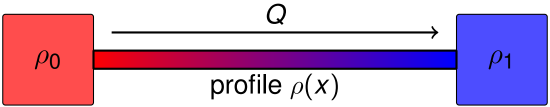

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$$\langle \text{final}|e^{it\mathbb{H}}|\text{initial}\rangle = \int dz_1 \ldots dz_n \langle \text{final}|e^{i\Delta t\mathbb{H}}|\underline{z_n}\rangle\langle\underline{z_{n-1}}|e^{i\Delta t\mathbb{H}}|\underline{z_{n-2}}\rangle \ldots$$
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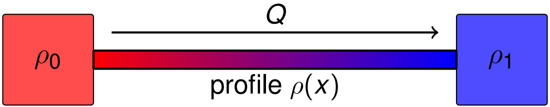

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$$\ldots \langle\underline{z_1}|e^{i\Delta t\mathbb{H}}|\text{initial}\rangle$$

$$= \int \mathcal{D}p\mathcal{D}q \ \exp\{i\tfrac{1}{\hbar}\underbrace{\mathcal{S}[p,q]}_{\text{action}}\}$$

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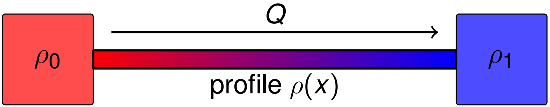

Using $SU(2)$ coherent states:

$$\langle \rho_{\mathrm{f}} | e^{t\mathbb{W}} | \rho_{\mathrm{i}} \rangle = \int_{\rho(0)=\rho_{\mathrm{i}}}^{\rho(t)=\rho_{\mathrm{f}}} \mathcal{D}\rho\mathcal{D}\hat{\rho}\ \exp\{L \underbrace{\mathcal{S}[\hat{\rho},\rho]}_{\text{action}}\}$$

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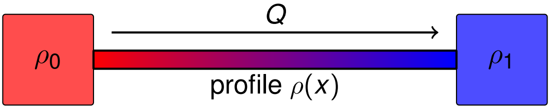

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$$\langle e^{-sQ} \rangle \sim \langle \rho_{\rm f} | e^{t\mathbb{W}_s} | \rho_{\rm i} \rangle = \int_{\rho(0)=\rho_{\rm i}}^{\rho(t)=\rho_{\rm f}} \mathcal{D}\rho\mathcal{D}\hat{\rho}\ \exp\{L \underbrace{\mathcal{S}_s[\hat{\rho},\rho]}_{\text{action}}\}$$

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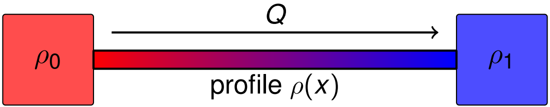

Same $\mathcal{S}_s[\hat{\rho}, \rho]$ as the MSR action of the Langevin evolution:

$$\partial_t \rho = -\partial_x \big[-\partial_x \rho + \xi \big]$$

$$\langle \xi(x,t)\xi(x',t')\rangle = \underbrace{\frac{1}{L}\rho(1-\rho)}_{\text{density-dependent}} \quad \delta(x'-x)\delta(t'-t)$$

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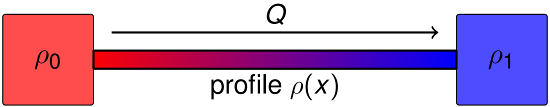

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One recovers the action of fluctuating hydrodynamics
[Spohn, Bertini De Sole Gabrielli Jona-Lasinio Landim]

$\psi(s)$: again [Appert, Derrida, VL, van Wijland, PRE **78** 021122]

Periodic boundary conditions
More general fluctuating hydrodynamics

$$\frac{1}{Lt}\langle Q\rangle \propto D(\rho) \qquad \text{(Fourier's law)}$$

$$\frac{1}{Lt}\langle Q^2\rangle_c = \sigma(\rho) \qquad \text{(For the SSEP, } \sigma = \rho(1-\rho)\text{)}$$

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Saddle point evaluation

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Correspondence between
(Gaussian) integration of small fluctuations
AND
discreteness of Bethe root repartition.

More general?

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More general?

Fluctuating hydrodynamics for quantum chains?

With a field [Appert, Derrida, VL, van Wijland, PRE **78** 021122]

Periodic boundary conditions
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Periodic boundary conditions
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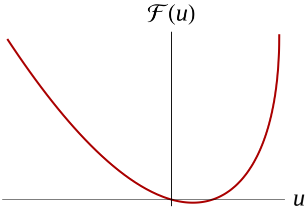

Dynamical phase transition
between
stationnary and non-stationnary
profiles

Outline

Microscopic approach [Imparato, VL, van Wijland, arXiv:0911.0564]

Large deviations of the current

$$\psi(s) = \max \mathrm{Sp}\, \mathbb{W}(s)$$

$$\mathbb{W}(s) = \overbrace{\sum_{1\leq k\leq L-1} \vec{S}_k \cdot \vec{S}_{k+1}}^{\text{invariant by rotation}}$$
$$+ \alpha \left[S_1^+ - (1-\hat{n}_1)\right] + \gamma \left[S_1^- - \hat{n}_1\right]$$
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Local transformation

$$\mathcal{Q}^{-1}\mathbb{W}(s)\mathcal{Q} = \sum_{1\le k\le L-1} \vec{S}_k \cdot \vec{S}_{k+1}$$
$$\underbrace{+ \alpha' \left[S_1^+ - (1-\hat{n}_1)\right] + \gamma' \left[S_1^- - \hat{n}_1\right] + \delta' \left[S_L^+ e^{s'} - (1-\hat{n}_L)\right] + \beta' \left[S_L^- e^{-s'} - \hat{n}_L\right]}_{\text{describes contact with reservoirs of same densities}}$$

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Local transformation Fluctuations@eq $\leftrightarrow$ Fluctuations@non-eq

$$\mathcal{Q}^{-1} \mathbb{W}(s) \mathcal{Q} = \sum_{1 \le k \le L-1} \vec{S}_k \cdot \vec{S}_{k+1} + \alpha' \left[S_1^+ - (1 - \hat{n}_1) \right] + \gamma' \left[S_1^- - \hat{n}_1 \right] \underbrace{+ \delta' \left[S_L^+ e^{s'} - (1 - \hat{n}_L) \right] + \beta' \left[S_L^- e^{-s'} - \hat{n}_L \right]}_{\text{describes contact with reservoirs of same densities}}$$

Macroscopic approach [Imparato, VL, van Wijland, **PRE** 80 011131]

$$\langle e^{-sQ}\rangle \sim \int \mathcal{D}\rho\mathcal{D}\hat{\rho}\ \exp\{L\,\mathcal{S}_s[\hat{\rho},\rho]\}$$

Fluctuations ϕ, $\hat{\phi}$ around the saddle

$$\rho(x,t) = \rho_c(x) + \phi(x,t) \qquad \hat{\rho}(x,t) = \hat{\rho}_c(x) + \hat{\phi}(x,t)$$

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Mapping of non-eq. fluctuations ϕ, $\hat{\phi}$ to eq. fluctuations ϕ', $\hat{\phi}'$

$$\phi(x,t) = (\partial_x \hat{\rho}_c)^{-1} \phi'(x,t) + (\partial_x \rho_c)^{-1} \hat{\phi}'(x,t)$$
$$\hat{\phi}(x,t) = (\partial_x \hat{\rho}_c) \hat{\phi}'(x,t)$$

Macroscopic approach

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$$\langle e^{-sQ}\rangle \sim \int \mathcal{D}\rho\mathcal{D}\hat{\rho}\ \exp\{L\,\mathcal{S}_s[\hat{\rho},\rho]\}$$

Fluctuations ϕ, $\hat{\phi}$ around the saddle

$$\rho(x,t) = \rho_c(x) + \phi(x,t) \qquad \hat{\rho}(x,t) = \hat{\rho}_c(x) + \hat{\phi}(x,t)$$

Mapping of non-eq. fluctuations ϕ, $\hat{\phi}$ to eq. fluctuations ϕ', $\hat{\phi}'$

$$\phi(x,t) = (\partial_x\hat{\rho}_c)^{-1}\phi'(x,t) + (\partial_x\rho_c)^{-1}\hat{\phi}'(x,t)$$

$$\hat{\phi}(x,t) = (\partial_x\hat{\rho}_c)\hat{\phi}'(x,t)$$

Large deviation function

$$\psi(s) = \underbrace{\frac{1}{L}\mu(s)}_{\text{saddle}} + \overbrace{\underbrace{\frac{D}{8L^2}\mathcal{F}\left(\frac{\sigma''}{2D^2}\mu(s)\right)}_{\text{fluctuations}}}^{\textbf{same } \mathcal{F} \textbf{ as at eq.}}$$

Macroscopic approach [Imparato, VL, van Wijland, **PRE** 80 011131]

Physical remark

In contact with reservoirs, neither SSEP nor KMP present a 'dynamical phase transition' ($\Leftarrow$ no singularity in $\psi(s)$).

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- start from $\mathbb{W}(s)$
- use coherent states to obtain a *spatially-discrete* action $S_s[\rho, \hat{\rho}]$
- solve the *spatially-discrete* stationary saddle-point equations

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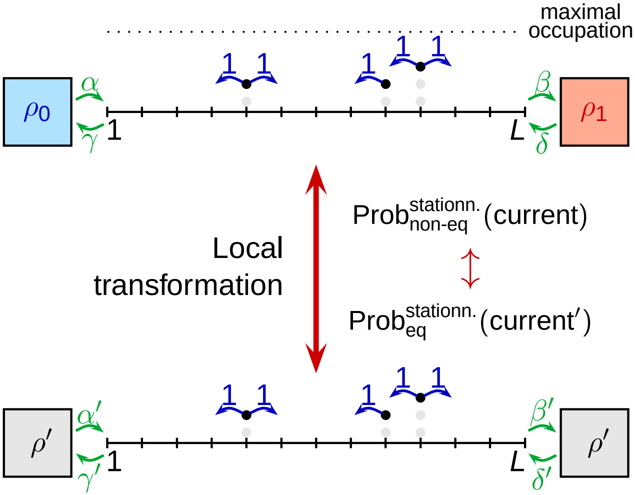
$\longrightarrow$ How does this translate in the Bethe Ansatz approach?

For the current

[Imparato, VL, van Wijland, **PRE** 80 011131]

Symmetric exclusion process



$$\text{Prob}^{\text{stationn.}}_{\text{non-eq}}(\text{current})$$

$$\updownarrow$$

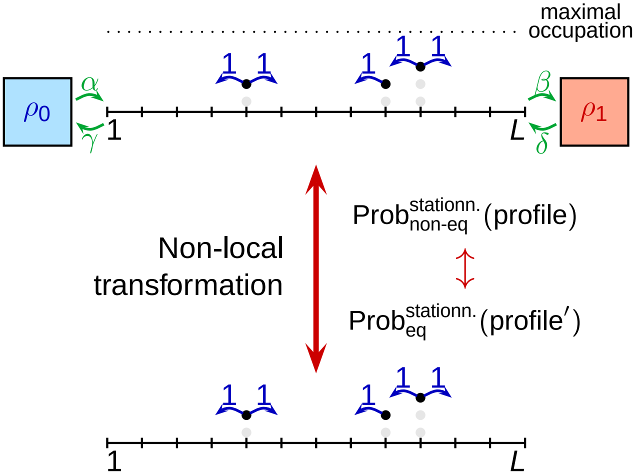
$$\text{Prob}^{\text{stationn.}}_{\text{eq}}(\text{current}')$$

System in equilibrium

For the density profile

[Tailleur, Kurchan, VL, JPA **41** 505001]

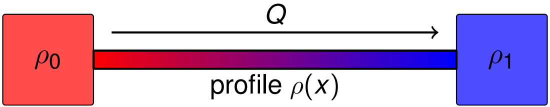
Symmetric exclusion process



System in equilibrium

Non-local mapping

[Tailleur, Kurchan, VL, JPA **41** 505001]

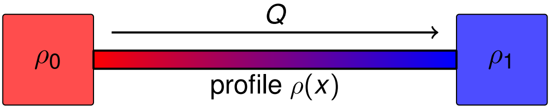


Boundary-driven transport model:

- long-range correlations
- breaking of time-reversal symmetry

Non-local mapping

[Tailleur, Kurchan, VL, JPA **41** 505001]



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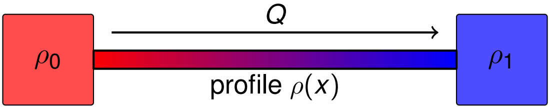

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$\longrightarrow$ Applies to non-equilibrium quantum chains?

Summary

Approach:

- operator formalism
- large deviation function

Extensions:

- fluctuating hydrodynamics
- saddle-point method, instantons
- integration of fluctuations (dynamical phase transition)

Open questions:

$\rightarrow$ Eq$\leftrightarrow$non-eq mapping in higher dimensions?
$\rightarrow$ More generic systems of interacting particle?
$\rightarrow$ Link btw the eq$\leftrightarrow$non-eq mappings?
$\rightarrow$ Crossover to KPZ? Other universal fluctuations?