Interfaces with short-range correlated disorder: what we learn from the Directed Polymer

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Lev Truskinovsky[(3,5)]

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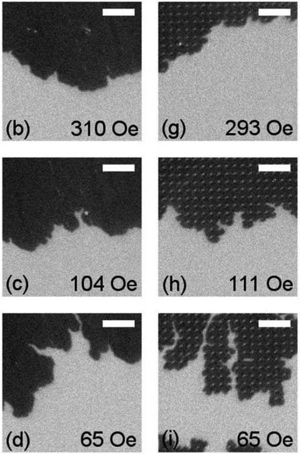
1D Interfaces

Interfaces in magnetic films



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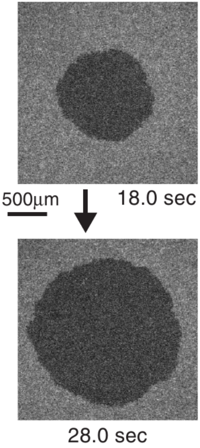
APL **94** 132504 (2009)

Large range of physical scales

Wide spectrum of phenomena

Growth in liquid crystals



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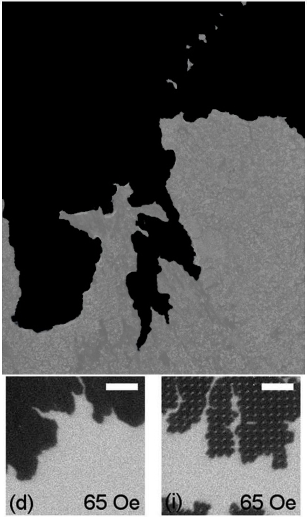

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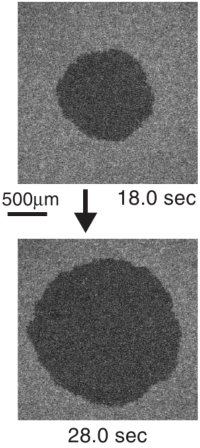

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Disordered elastic systems

- Elasticity: tends to flatten the interface

$$\mathcal{H}^{\text{el}} = \frac{c}{2} \int dz \, \big(\nabla u(z)\big)^2 \qquad \text{[Short-range]}$$

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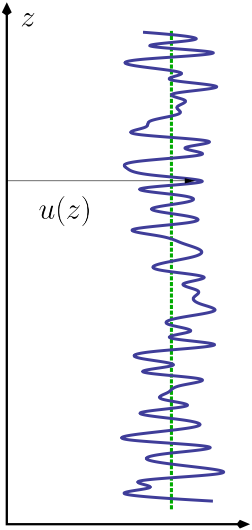
$$\mathcal{H}_V^{\text{dis}} = \int dz \, V\big(u(z), z\big)$$



Competition btw "order" and "disorder"

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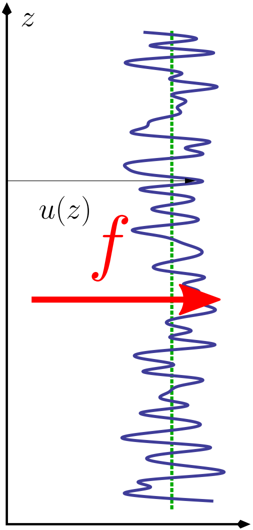
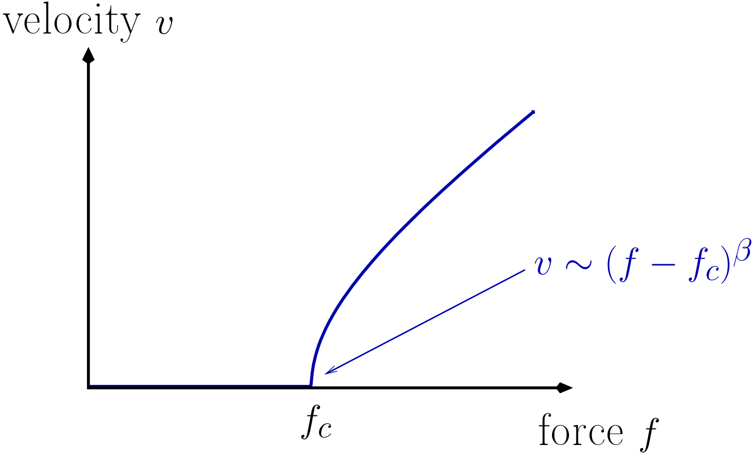


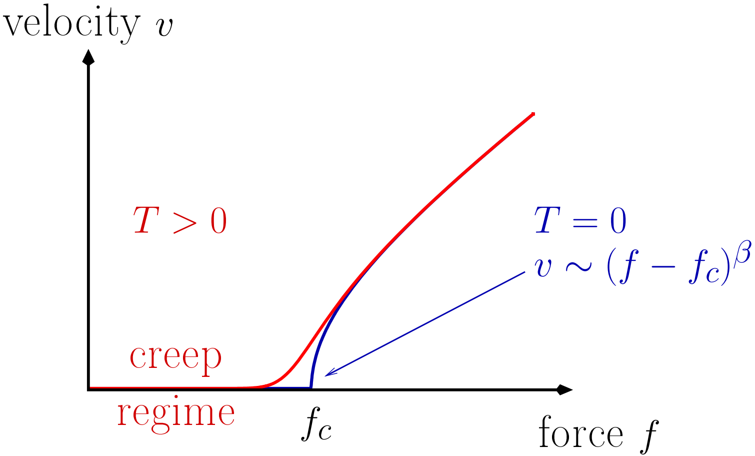
Depinning transition @ zero temperature

threshold force f_c



Depinning transition @ finite temperature

thermal rounding
creep regime



Uncorrelated disorder:

$$\overline{V(z,x)\,V(z',x')} = D\,\delta(z'-z)\delta(x'-x)$$

Correlated disorder on a lengthscale ξ:

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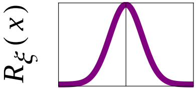

scaling as $R_\xi(x) = \frac{1}{\xi}\,R_{\xi=1}(x/\xi)$

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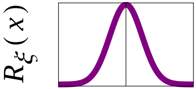
Can ξ play a role at lengthscales $\gg \xi$?



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Study 1D models with correlated disorder ($\xi > 0$)

1. Static properties & creep regime ($T > 0$ and $T \to 0$)
   short-range elasticity
   $\longrightarrow$ **Identification of lengthscales**
   [with Elisabeth Agoritsas, Thierry Giamarchi]

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   $\longrightarrow$ **Non-equilibrium velocity** & equilibrium free-energy
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3. Depinning force f_c $(T = 0)$
   long-range elasticity
   $\longrightarrow$ **Role of disorder correlator**
   [with Vincent Démery, Alberto Rosso]

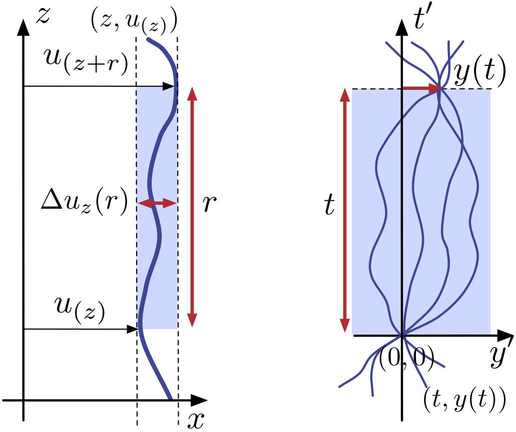
1D Interface in the Directed Polymer (DP) language

[Step n°1]

- No bubbles
- No overhangs
- Interface lengthscale r

$\updownarrow$

DP 'time' t



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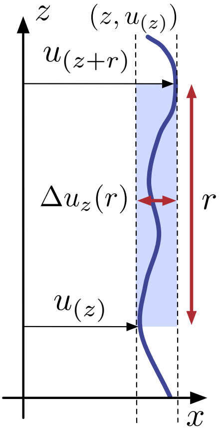

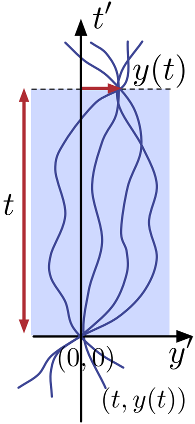

working at fixed 'time' t $\iff$
integration of fluctuations at scales smaller than t

lengthscale $\equiv$ time duration

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- Ingredients up to now:

elastic constant c disorder potential $V(t,y)$ $\underbrace{\text{temperature } T}_{\text{trajectory weight } \propto e^{-\mathcal{H}_V/T}}$

Questions

- Nature of fluctuations of $y(t)$
  - ⋆ $V(t,y) \equiv 0$: *diffusive* ($y \sim t^{1/2}$), **Edwards-Wilkinson** (EW)
  - ⋆ $V(t,y) \not\equiv 0$: *super-diffusive* ($y \sim t^{2/3}$), **Kardar-Parisi-Zhang** (KPZ)

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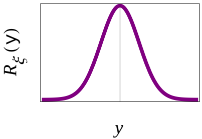
  zero mean, Gaussian, $\overline{V(t,y)V(t',y')} = D\,\delta(t'-t)R_\xi(y'-y)$



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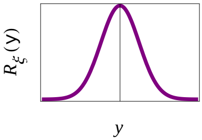

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- Summary of ingredients:

  elastic constant c temperature T disorder | amplitude D, corr. length ξ

Free-energy fluctuations [Step n°2&3]

- Partition function Z_V *vs.* Free-energy F_V

$$Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t') e^{-\frac{1}{T}\mathcal{H}_V[y(t'),t]} \qquad Z_V(t,y) = \exp\left\{-\frac{1}{T}F_V(t,y)\right\}$$

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- **Statistical Tilt Symmetry**

$$F_V(t,y) = \underbrace{c\frac{y^2}{2t} + \frac{T}{2}\log\frac{2\pi Tt}{c}}_{\substack{\text{thermal contribution} \\ F_{V\equiv 0}}} + \underbrace{\overline{F}_V(t,y)}_{\substack{\text{disorder} \\ \text{contribution}}} \qquad \textbf{(STS)}$$

- **Tilted** KPZ equation for $\overline{F}_V(t,y)$

$$\partial_t \overline{F}_V + \frac{y}{t}\partial_y \overline{F}_V = \frac{T}{2c}\partial_y^2 \overline{F}_V - \frac{1}{2c}\left[\partial_y \overline{F}_V\right]^2 + V(t,y)$$

Non-linear, additive noise, $\overline{F}_V(0,y) \equiv 0$: "simple" initial cond.

Known results $@\xi = 0$ $[\iff T \to \infty \; @\xi > 0]$

- **Central tool**: 2-point correlation function

$$\overline{R}(t, y_2 - y_1) = \overline{\partial_y \overline{F}_V(t, y_1) \partial_y \overline{F}_V(t, y_2)}$$

- **Infinite-'time' limit** (steady state)

$\overline{F}(t = \infty, y)$ distributed as a Brownian Motion

i.e. : $Prob\big[\,\overline{F}(t = \infty, y)\big]$ Gaussian, of correlator:

$$\overline{R}(t = \infty, y) = \widetilde{D}_{\xi=0}\, \delta(y) \quad \text{with} \quad \boxed{\widetilde{D}_{\xi=0} = \frac{cD}{T}}$$

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- **Roughness** function $B(t)$ [variance of end-point fluct.]

$$B(t) = \overline{\langle y(t)^2 \rangle} = \overline{\frac{\int dy\, y^2 Z_V(t,y)}{\int dy\, Z_V(t,y)}}$$

$$\boxed{B(t) \sim \big[\widetilde{D}_{\xi=0} / c^2\big]^{2/3} t^{4/3}} \quad \text{as } t \to \infty$$

Effective model @$\xi > 0$ & numerical results

$\xi > 0$ not obtained from perturbation of $\xi = 0$

- **Distribution** of free-energy

  scales closely to the $\xi = 0$ case

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- **Distribution** of free-energy
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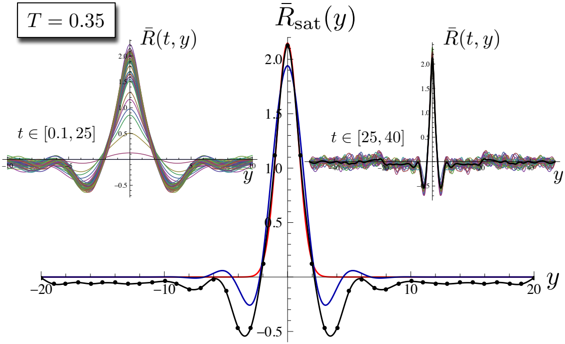
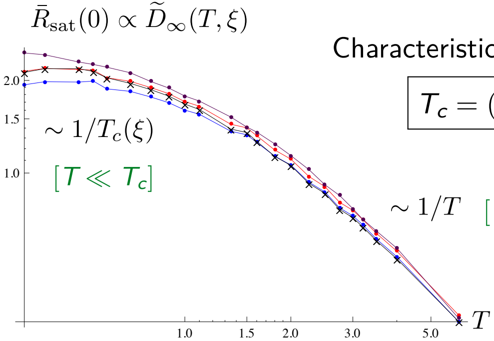
- **2-point** correlation function of amplitude $\widetilde{D}$

$$\overline{R}_{\text{sat}}(y) = \lim_{t\to\infty} \overline{R}(t,y) \simeq \widetilde{D} R_\xi(y)$$



High- and low-temperature regimes



Characteristic temperature

$$T_c = (\xi c D)^{1/3}$$

- (Advanced) **scaling** analysis

$T \ll T_c$	$T \gg T_c$
one optimal trajectory	many trajectories
$\widetilde{D} = \dfrac{cD}{T_c}$	$\widetilde{D} = \dfrac{cD}{T}$

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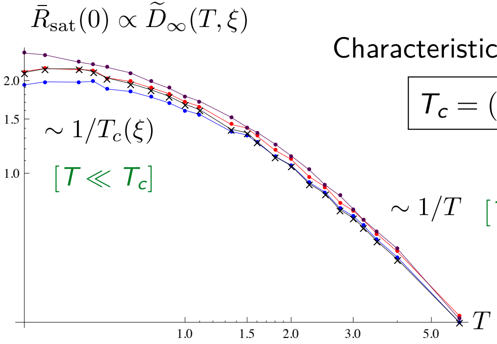

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ξ plays a role at all scales

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Lengthscales & dynamics

- **Geometry** of interface $\longleftrightarrow$ Directed Polym. **free-energy** fluctuat.
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  - $\star$ understanding of 'time'- (i.e. length) multiscaling

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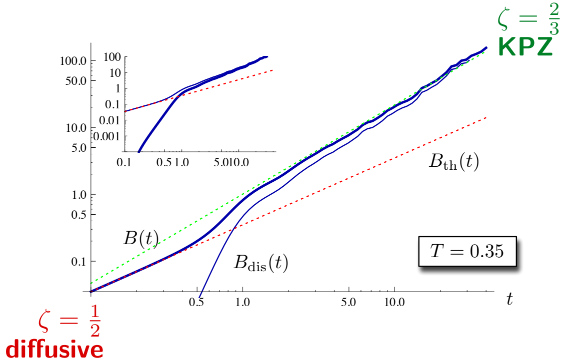

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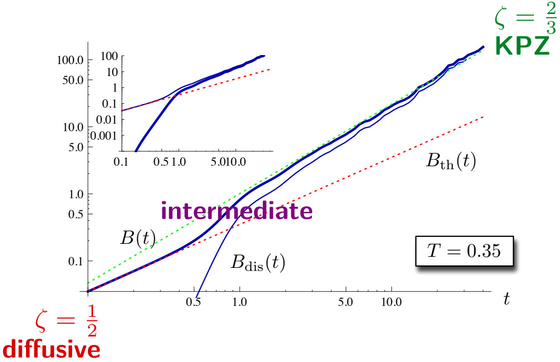

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- Interpretation in other '**incarnations**' of the KPZ class
  - $\star$ growth interfaces with $F(t,y) =$ height at (real) time t
  - $\star$ experimental probe of the importance of ξ
  - $\star$ through replicæ: **1D quantum bosons** with softened delta attractive interaction

Creep law [in progress]

[with R García-García, E Agoritsas, D Vandembroucq, L Truskinovsky]

- **Creep law**: non-linear response to small force

$$\text{velocity} \sim \exp\Big\{ -\Big[\frac{\overbrace{\text{critical force}}^{\text{depends on } c,\, D,\, T,\, \xi}}{\text{force}}\Big]^{1/4}\Big\}$$

- Very well verified numerically and experimentally
- Usual **energetic arguments**: self-inconsistent ($\zeta = \frac{5}{6} \neq \frac{2}{3}$)

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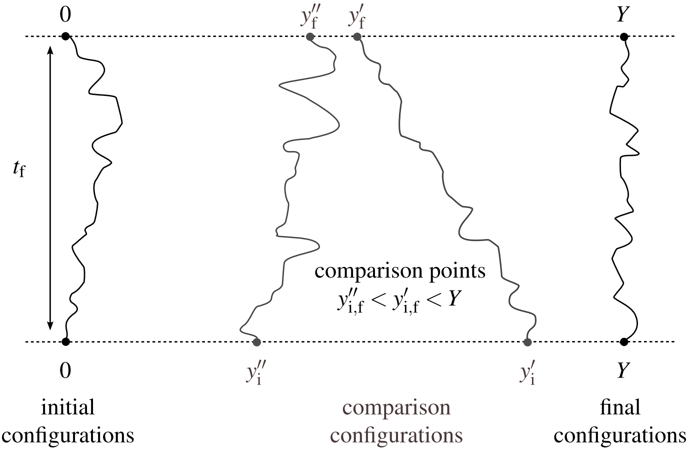
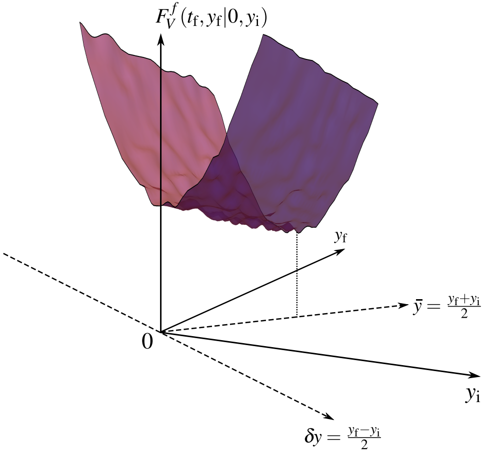
- Effective model:
  - ⋆ *Equilibrium* drifted polymer in a tilted potential
  - ⋆ generalized STS for its free-energy F [linear response works for F!]
  - ⋆ quasi-static picture: focus on extremities

Effective picture

$$\text{velocity} \sim \frac{1}{\text{mean first passage time}}$$



Effective potential seen by the polymer extremities



Saddle-point analyis (after well-chosen scaling)
$\rightarrow$ **creep law**, finite-size corrections

Depinning transition [with V Démery, A Rosso]

[following V Démery, L Ponson, A Rosso EPL **105** 34003 (2014)]

Equation of evolution [long-range elasticity]

$$\partial_t u(z,t) = f_{\mathrm{el}}[u(\cdot,t)](z) - \sigma \partial_u V\big(z, u(z,t)\big) + f$$

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Method: $[T=0]$

- ⋆ add a **confining** potential moving at **constant velocity**
- ⋆ perform a 1st order **perturbation** in disorder
- ⋆ obtain force(velocity) $\Longrightarrow$ send velocity to zero to get f_c
- ⋆ send confining potential to zero

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Disorder correlations: [for the disorder-induced *random force*]

$$\overline{\partial_z V(z,x) \partial_z V(z',x')} = \Delta_u(z'-z)\Delta_x(x'-x)$$

Critical force

Result:

$$f_c = \frac{\sigma^2 \widetilde{\Delta}_x(0)}{4\pi c} \int dk_u \, |k_u| \widetilde{\Delta}_u(k_u) \qquad (\sigma \to 0)$$

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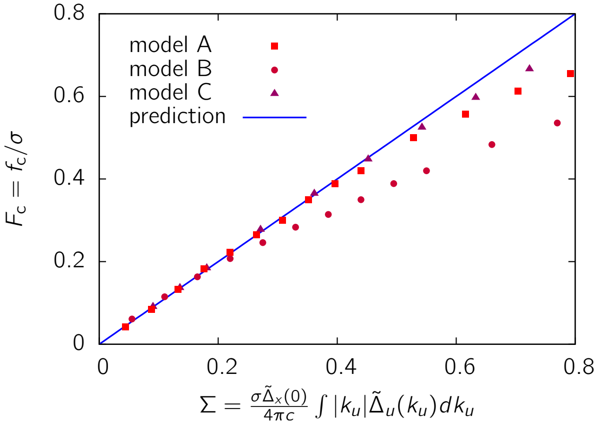

Thank you for your attention!

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  - soon on Arxiv