

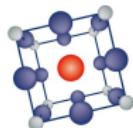
# Depinning transition for domain walls with an internal degree of freedom

Vivien Lecomte<sup>1</sup>, Stewart Barnes<sup>1,2</sup>, Jean-Pierre Eckmann<sup>3</sup>,  
Thierry Giamarchi<sup>1</sup>

<sup>1</sup>Département de Physique de la Matière Condensée, Genève

<sup>2</sup>Physics Department, University of Miami

<sup>3</sup>Département de Physique Théorique et Section de Mathématiques, Genève



**MaNEP**  
SWITZERLAND

Nancy – 14th May 2009



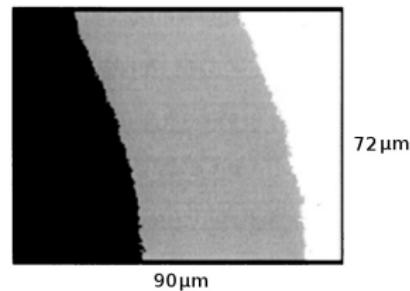
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# An example

## Interfaces in magnetic films



from Lemerle *et al.*, PRL **80** 849 (1998)

# Outline

## ① Interface Physics

- Systems
- Depinning transition
- Experiments

## ② Depinning with internal degree of freedom

- Modelisation
- Dynamics

# Interfaces

## Large range of physical scales

- magnetic/ferroelectric domain walls
- growth interfaces
- contact line
- crack propagation

## Common underlying description?

- Statics
  - fluctuations, roughness
- Non-equilibrium dynamics
  - motion of the interface
- Nature, role of disorder

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- Non-equilibrium **dynamics**
  - motion of the interface
- Nature, role of **disorder**

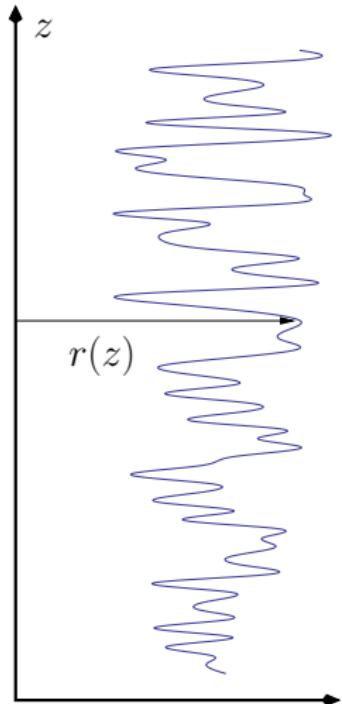
# Disordered elastic systems

- Elasticity: tends to flatten the interface

$$\frac{c}{2} \int dz (\nabla r(z))^2$$

- Disorder: tends to bend it

$$\int dz V(r(z), z)$$



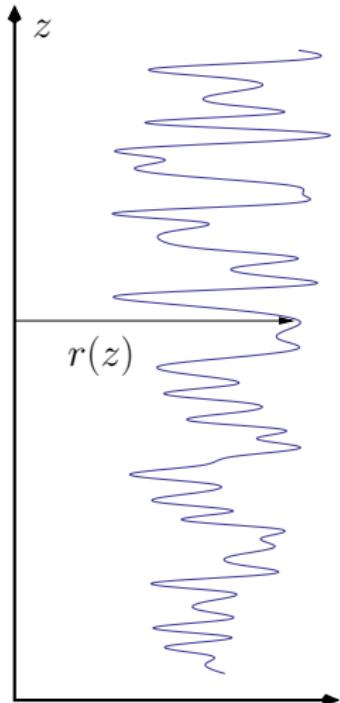
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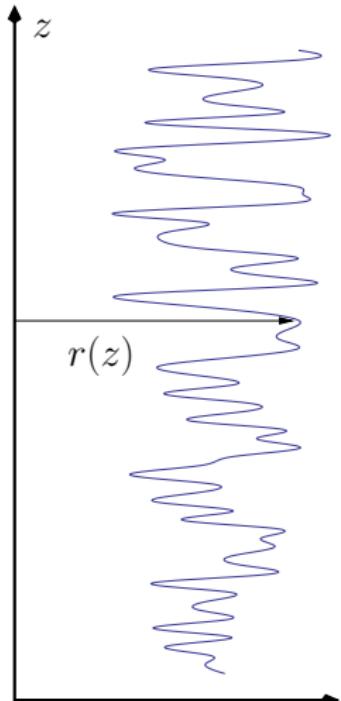
# Disordered elastic systems

- Elasticity: tends to **flatten** the interface

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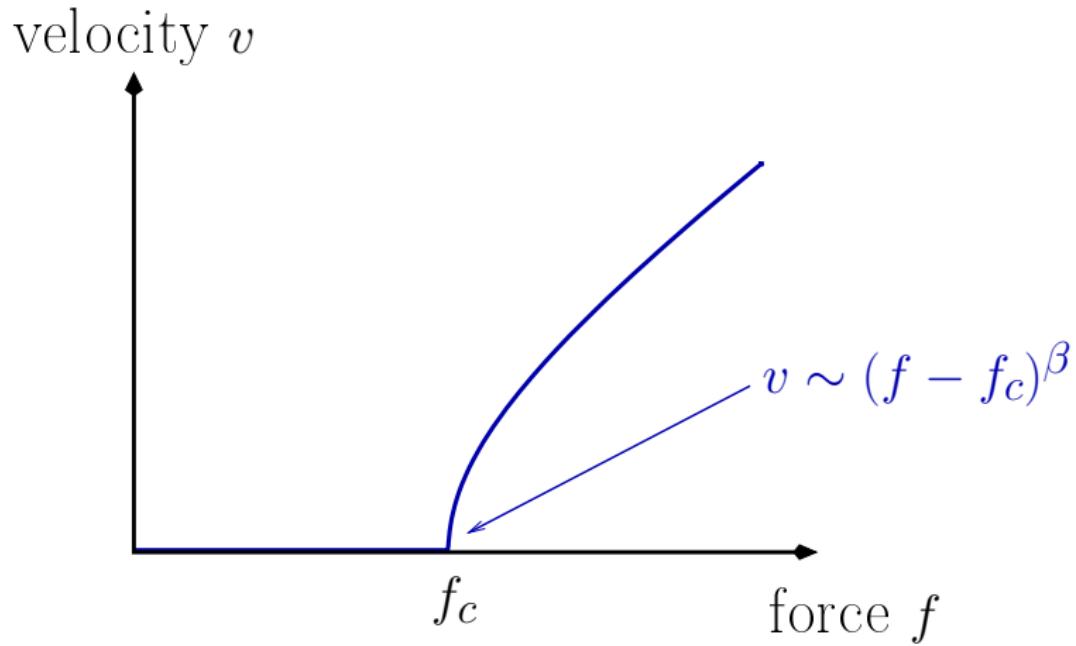
Competition btw “order” and “disorder”

# Is $r(z)$ containing enough?

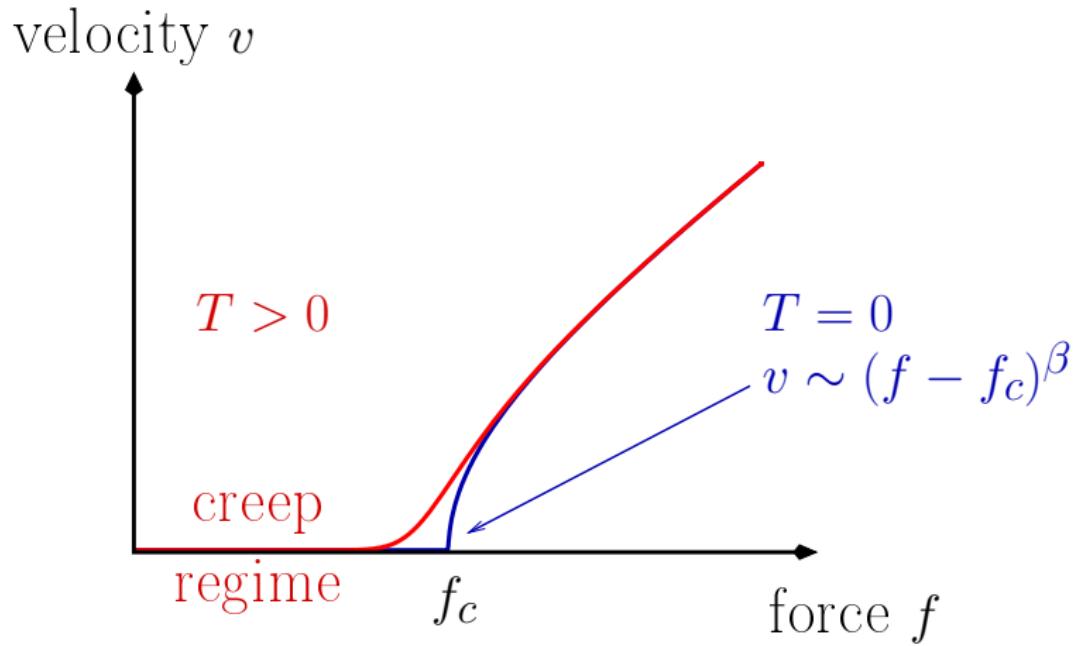
# Is $r(z)$ containing enough?

→ Have a look to the dynamics in simple examples.

# Depinning transition @ zero temperature



# Depinning transition @ finite temperature

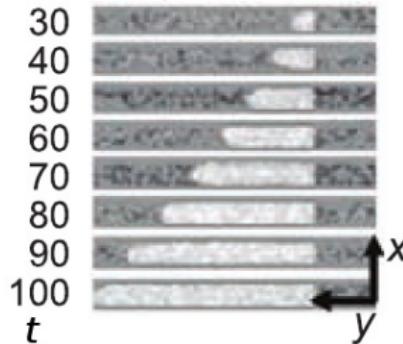


# Comparison with experiment: ferromagnetic wire

$$v(f) \sim \exp \left[ -\frac{U_c}{T} \left( \frac{f_c}{f} \right)^\mu \right] \quad (\text{creep})$$

	Field drive		Current drive	
	$\mu^*$	$\sigma^*$	$\mu$	$\sigma$
Experiment	$1.2 \pm 0.1$	$1.4 \pm 0.1$	$0.33 \pm 0.06$	$2.0 \pm 0.2$
Theory	1.0	1.5	0.5	1.25

from Yamanouchi *et al.*, Science 317 1726 (2007)



# Outline

## ① Interface Physics

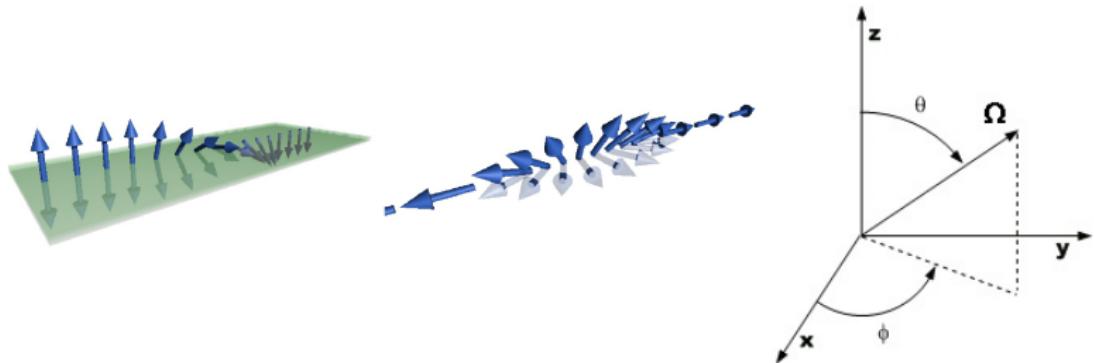
- Systems
- Depinning transition
- Experiments

## ② Depinning with internal degree of freedom

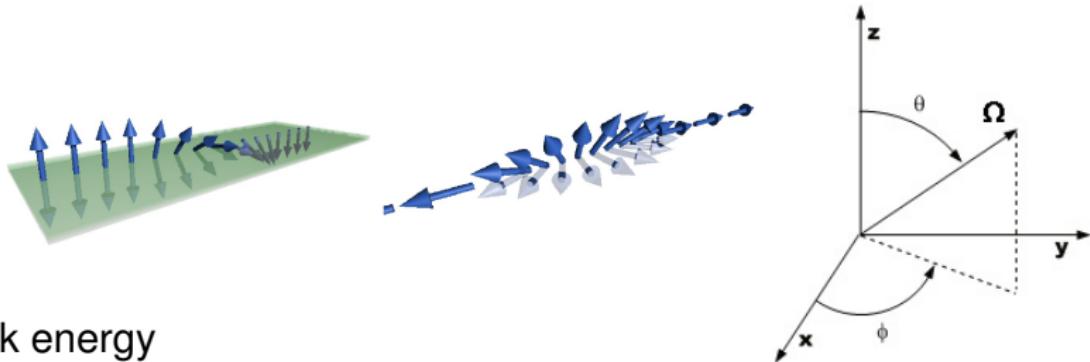
- Modelisation
- Dynamics



# Bulk model



# Bulk model



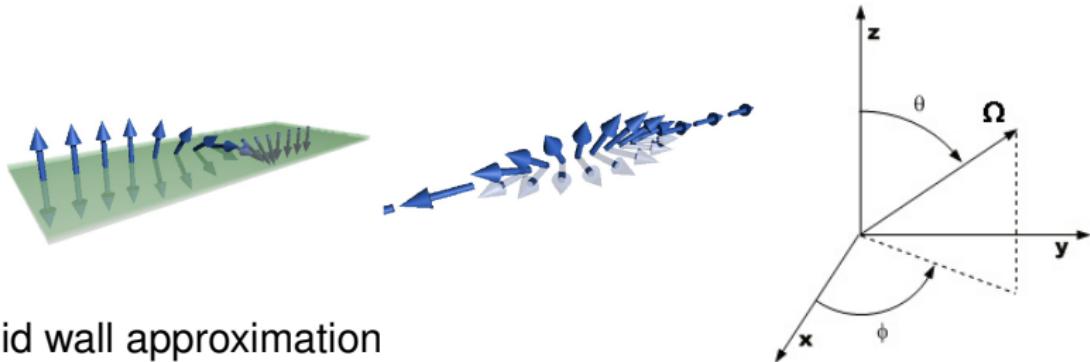
- Bulk energy

$$E = \int d^d x \left\{ J [(\nabla \theta)^2 + \sin^2 \theta (\nabla \phi)^2] + K \sin^2 \theta + K_{\perp} \sin^2 \theta \cos^2 \phi \right\}$$

- Equation of motion

$$(\partial_t + \mathbf{v}_s \cdot \nabla) \Omega = \Omega \times \left( \frac{\delta E}{\delta \Omega} + \mathbf{f} + \boldsymbol{\eta} \right) - \Omega \times (\alpha \partial_t + \beta \mathbf{v}_s \cdot \nabla) \Omega$$

# Bulk model



- Rigid wall approximation

$$\begin{aligned} \alpha \partial_t r - \partial_t \phi &= \underbrace{-\cos \kappa r}_{\text{pinning}} + \underbrace{f}_{\text{external}} + \eta_1 \\ \alpha \partial_t \phi + \partial_t r &= -\frac{1}{2} K_\perp \sin 2\phi + \eta_2 \end{aligned}$$

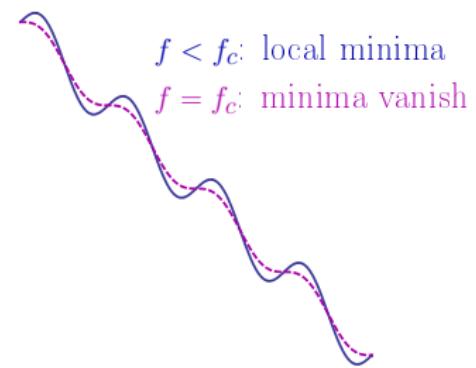
- Effective model:

**Position  $r(t)$  coupled to phase  $\phi(t)$ .**

# Depinning @

Large  $K_{\perp}$ :  $\phi$  decouples from  $r$

$$\alpha \partial_t r = f - \cos \kappa r$$

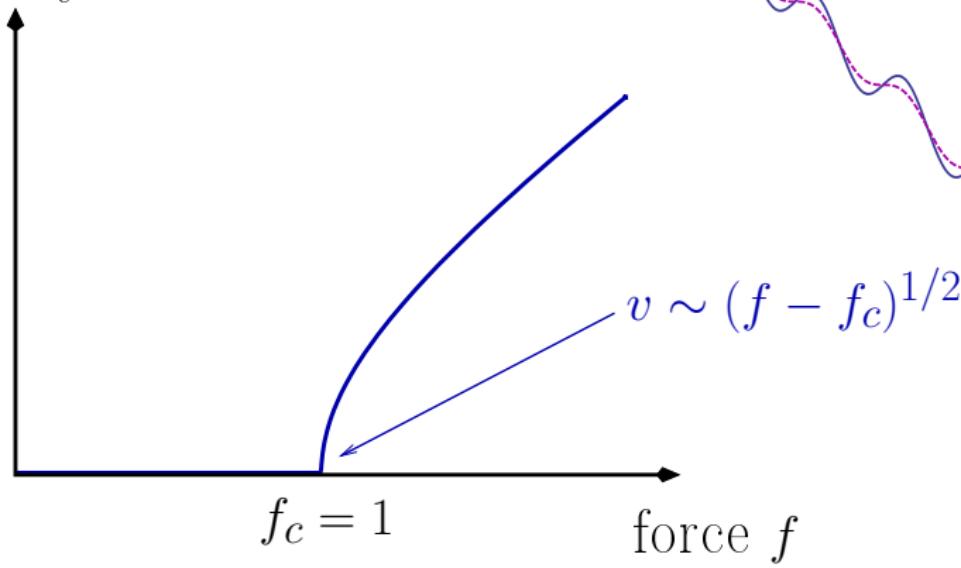


# Depinning @ zero temperature

Large  $K_{\perp}$ :  $\phi$  decouples from  $r$

$$\alpha \partial_t r = f - \cos \kappa r$$

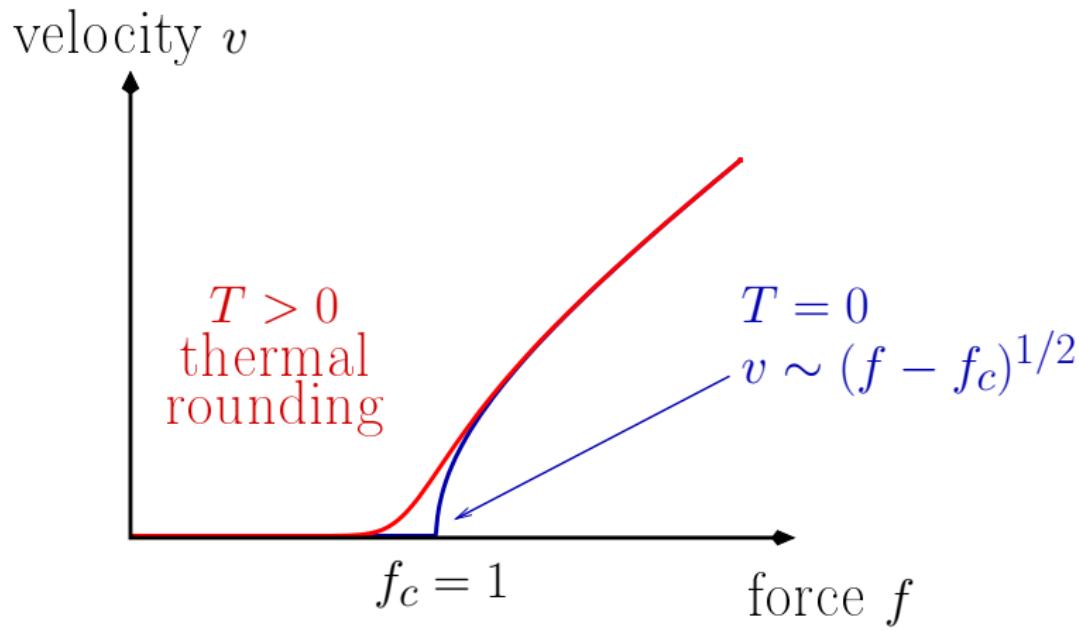
velocity  $v$



# Depinning @ finite temperature

Large  $K_{\perp}$ :  $\phi$  decouples from  $r$

$$\alpha \partial_t r = f - \cos \kappa r + \eta$$



# Depinning @ zero temperature

Smaller  $K_{\perp}$ :  $\phi$  matters

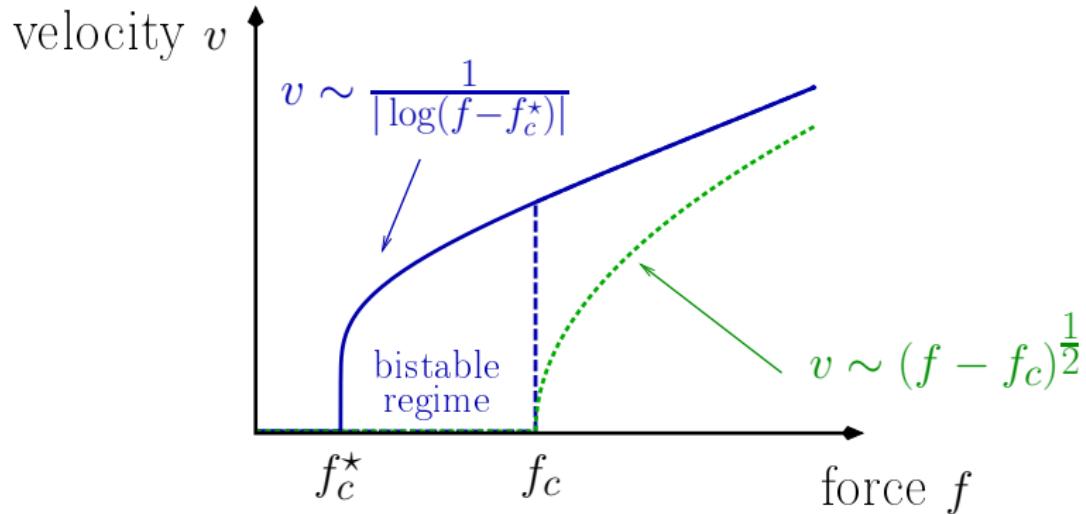
$$\alpha \partial_t r - \partial_t \phi = f - \cos \kappa r$$

$$\alpha \partial_t \phi + \partial_t r = -\frac{1}{2} K_{\perp} \sin 2\phi$$

# Depinning @ zero temperature

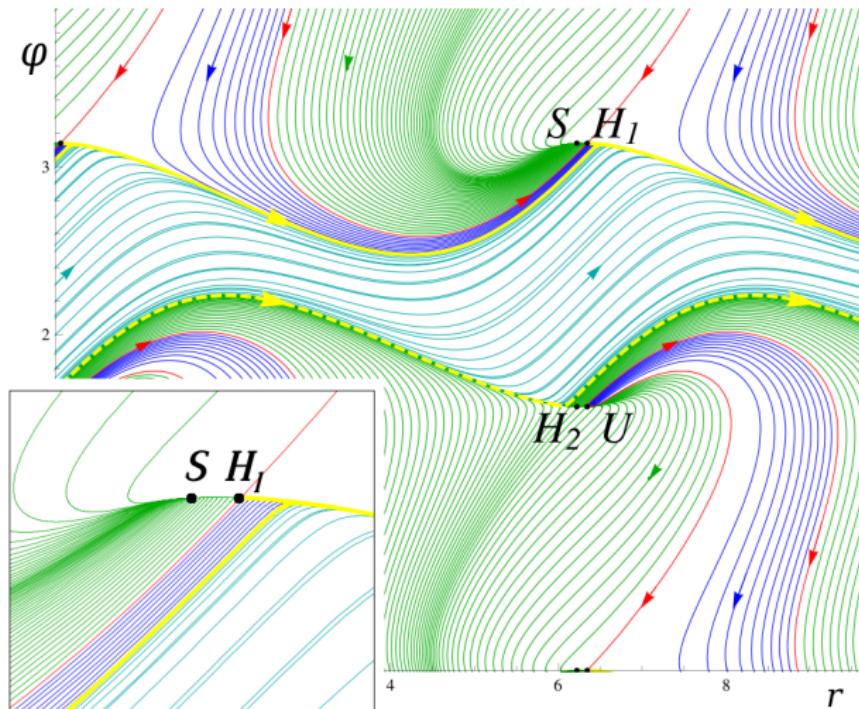
Smaller  $K_{\perp}$ :  $\phi$  matters

- Dramatic change in the depinning law:  $v \sim \frac{1}{|\log(f-f_c^*)|}$



- Depinning at **lower** critical force:  $f_c^* < f_c$
- Bistability

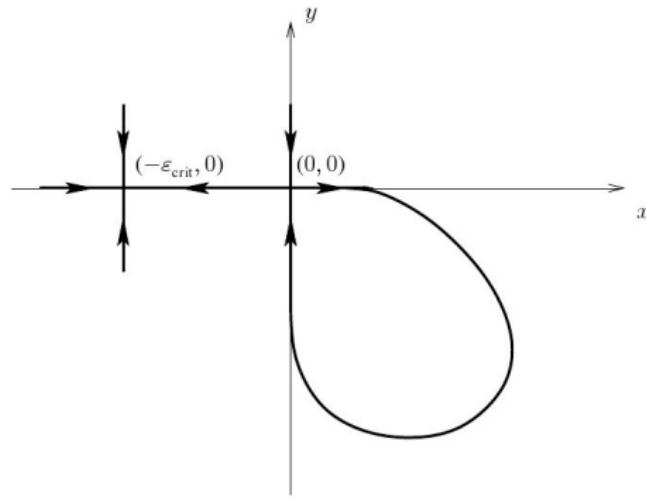
# Phase space



In the bistable regime ( $f_c^* < f < f_c$ )

# Phase space

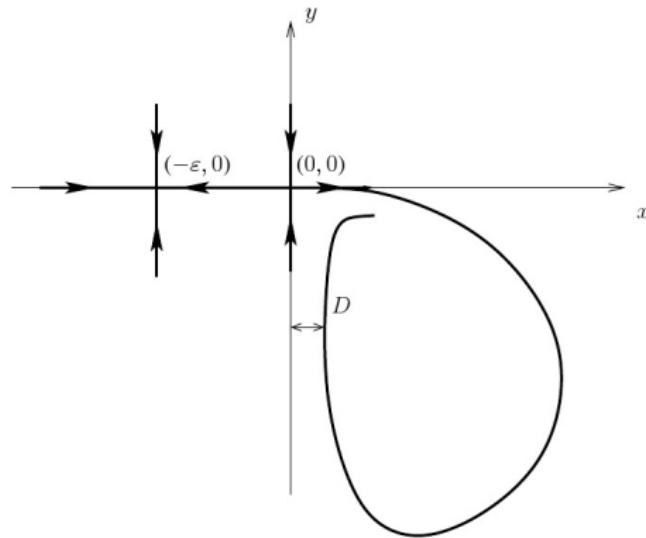
Homoclinic bifurcation:



$$f = f_c^*$$

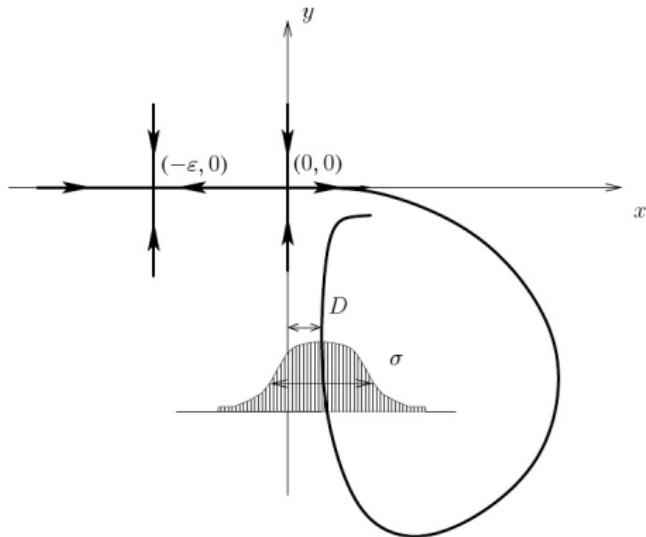
# Phase space

Homoclinic bifurcation:



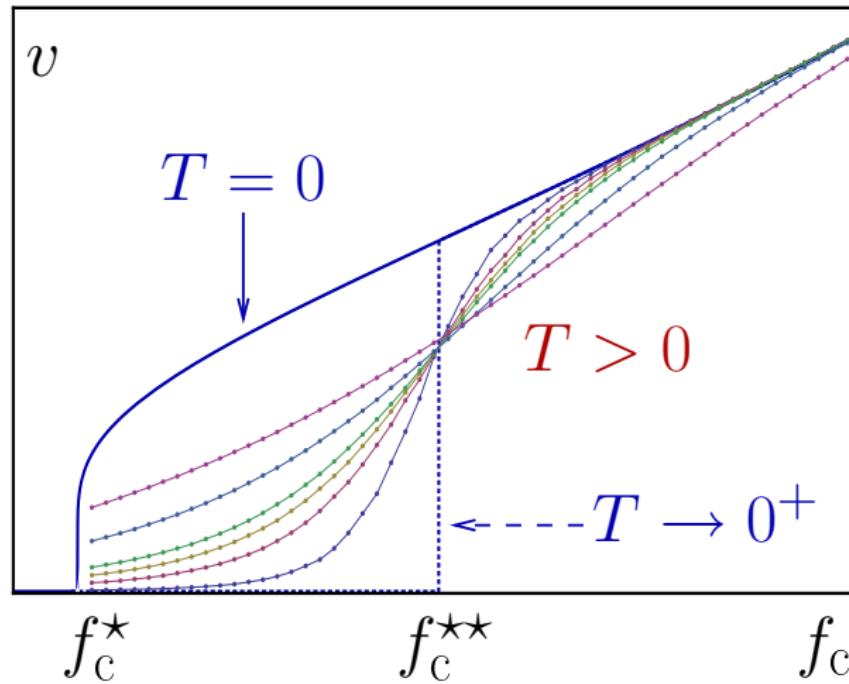
$$f > f_c^*$$

# Finite temperature



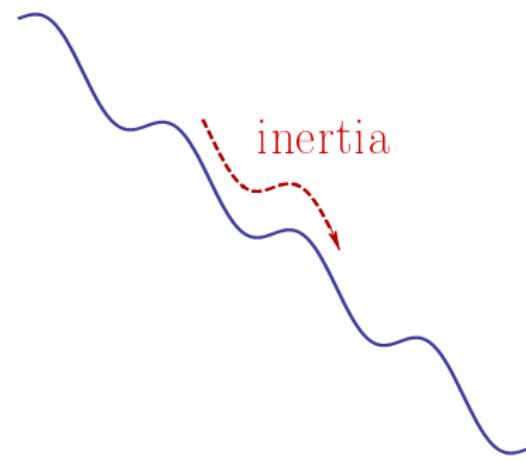
$$\text{escape time} \sim \underbrace{\exp\left(\frac{\epsilon^3}{T}\right)}_{\text{Arrhenius}} \underbrace{\exp\left(-\frac{A}{T}(\epsilon - \epsilon_c)^2\right)}_{\text{Trapping probability}}$$

# Finite temperature



Force-velocity characteristics

# Analogy

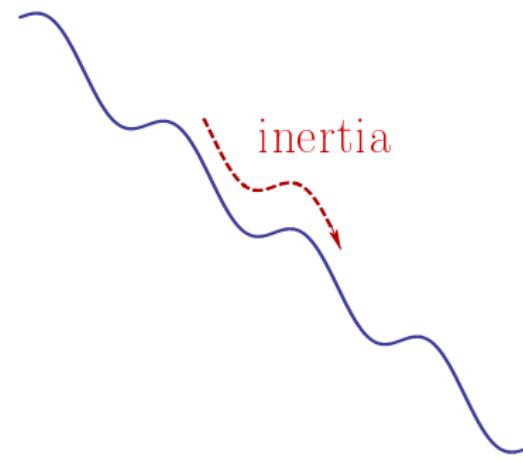


The phase  $\phi$  plays the role of a velocity:

inertia helps to cross barriers

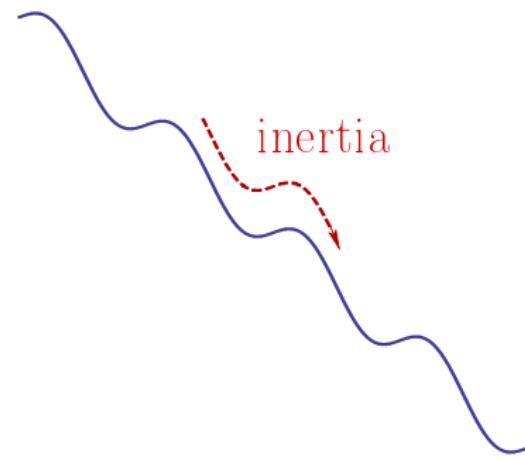
[see also Risken chap.11]

# Analogy



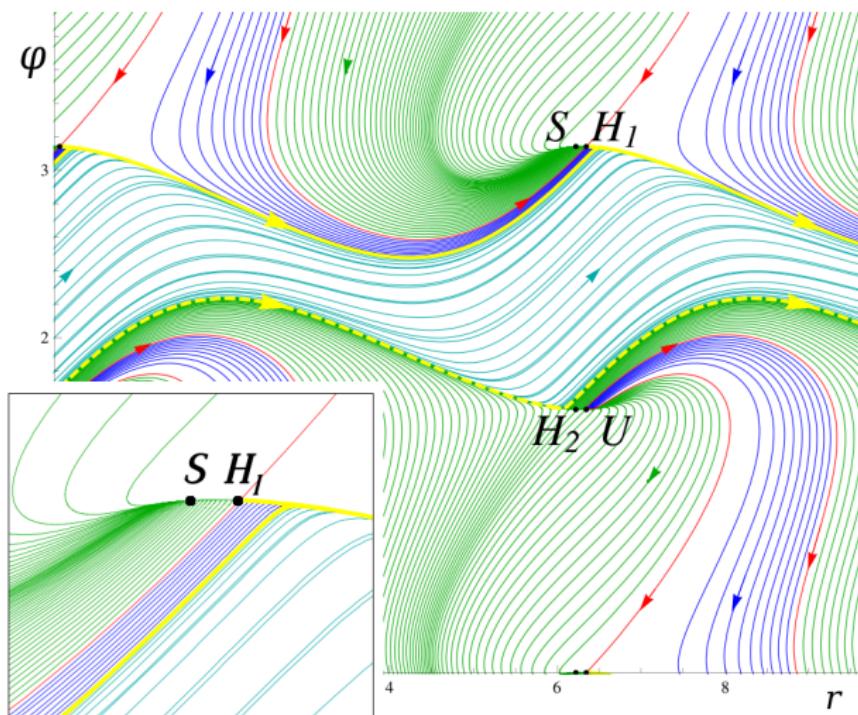
BUT ...

# Analogy

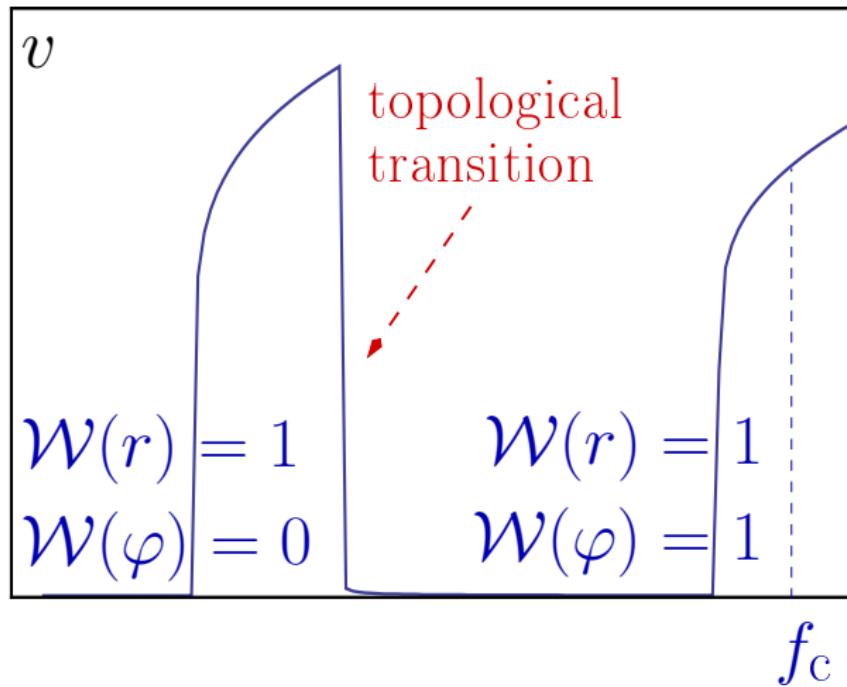


the velocity is **unbounded** **WHEREAS**  $\phi$  is **bounded** and **periodic**

# Topological transition



# Topological transition

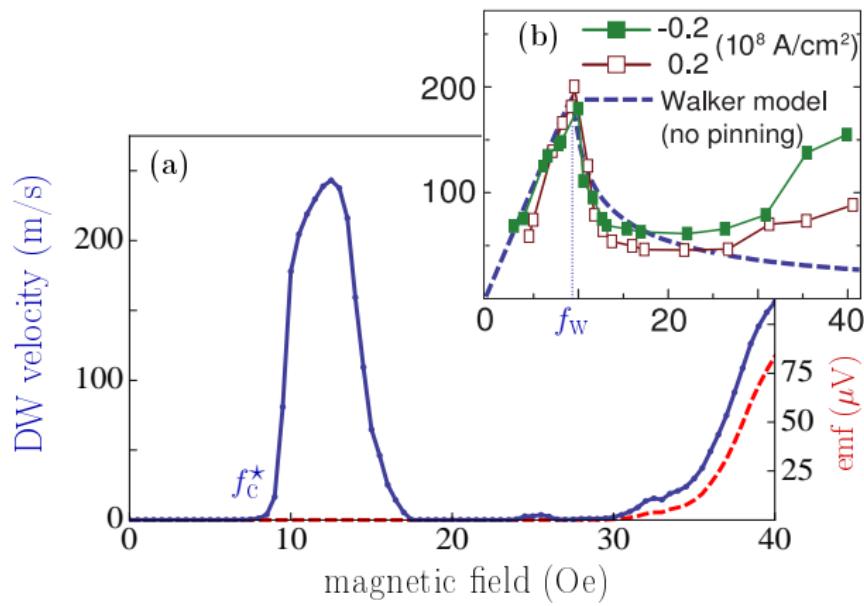


Successive regimes characterized by winding numbers  $\mathcal{W}$

# Experiment

# SPINTRONICS

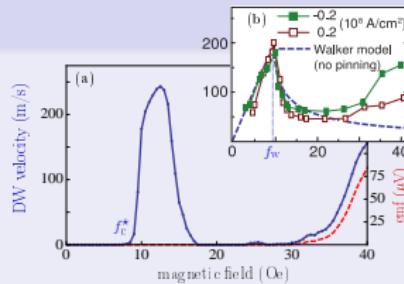
experiment from Parkin *et al.*, Science **320** 190 (2008)



# Outlook

## Internal degree of freedom

- unusual depinning law
- bistability
- non-monotonous  $v(f)$  at finite T
- link with experiments



## Perspective

- Current driven wall
  - Interface with elasticity
- modified creep law?

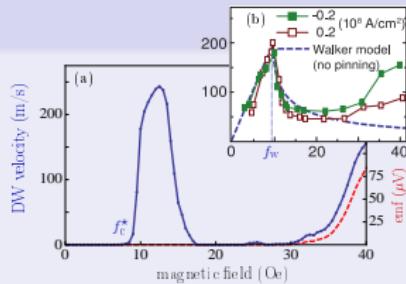
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periodic patterning

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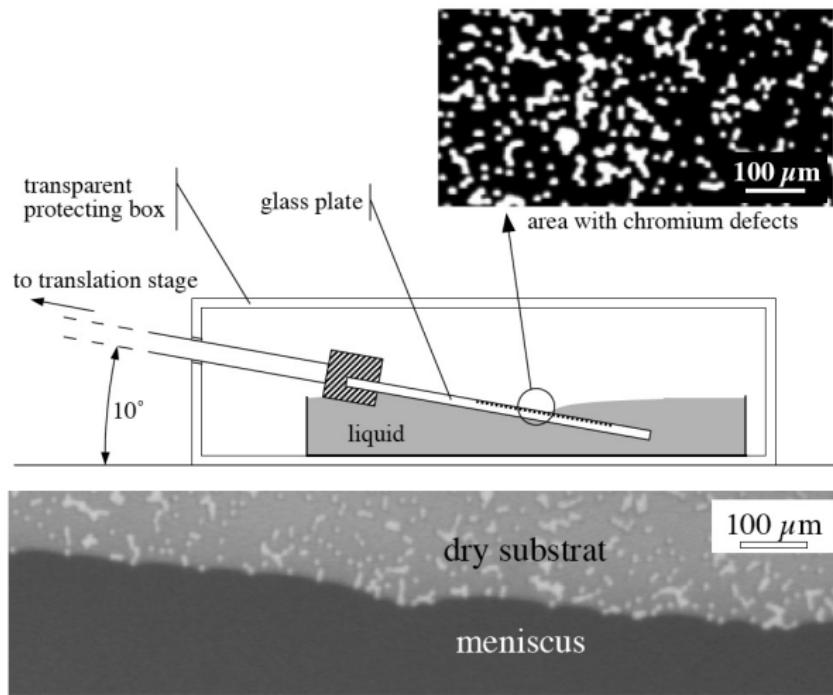
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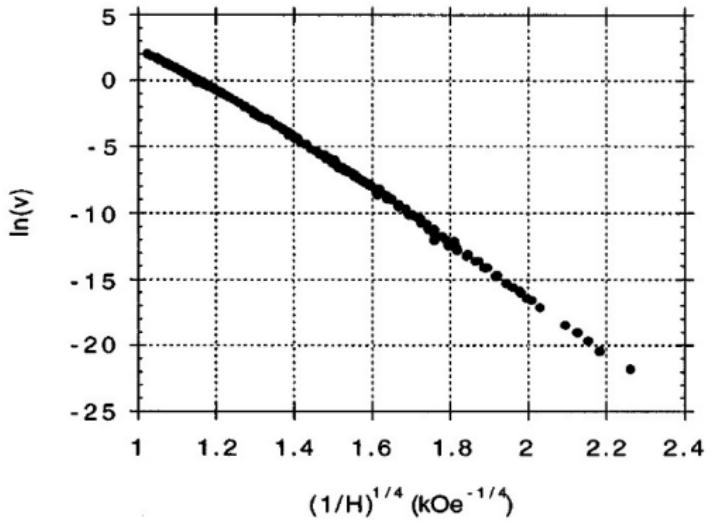
# Contact line of a fluid



from Moulinet, Guthmann and Rolley, *Eur. Phys. J. E*, **8** 437 (2002)

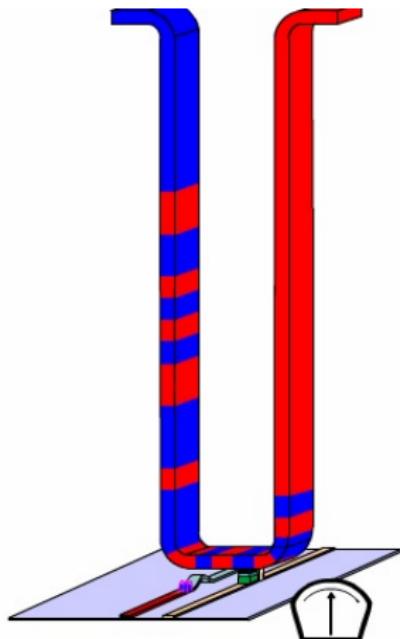
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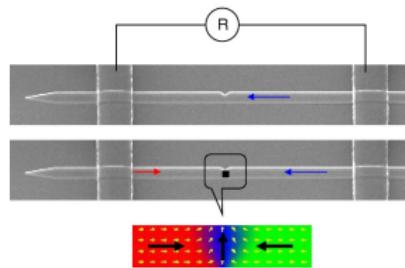


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SPINTRONICS



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