STATICS AND DYNAMICS OF INTERFACES IN CORRELATED DISORDER: WHAT WE LEARN FROM THE DIRECTED POLYMER

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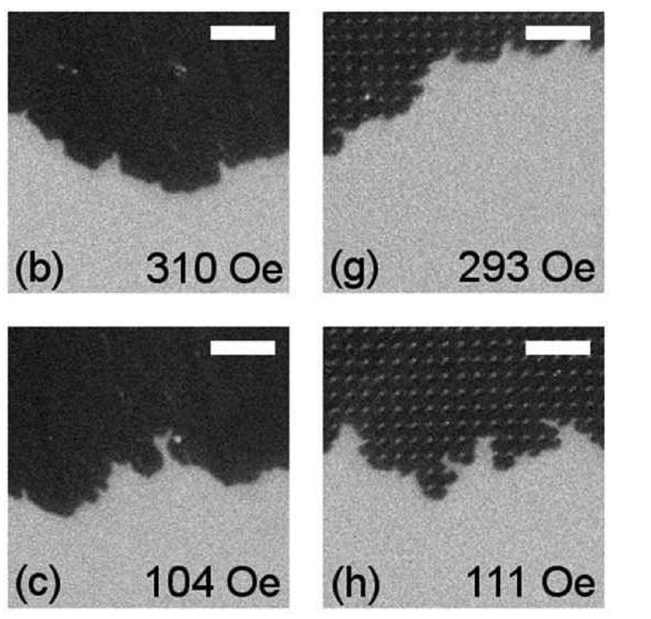
Abstract

One-dimensional boundary interfaces between different phases are described at macroscopic scales by a rough fluctuating line, whose geometrical properties are dictated by the disorder in the underlying medium, by the temperature of the environment, and by the elastic properties of the line.
A widely used and successful model is the **Directed Polymer** (DP) in a random medium, pertaining to the Kardar-Parisi-Zhang (**KPZ**) universality class. Much is known for this continuous model when the disorder is uncorrelated, and it has allowed to understand the static and dynamical features of experimental systems ranging from magnetic interfaces to liquid crystals.
Short-range correlations in the disorder at a lengthscale $\xi > 0$ modify the uncorrelated (*i.e.* zero ξ) picture in a non-obvious way. If the geometrical fluctuations are still described by the celebrated 2/3 KPZ exponent, characteristic amplitudes are however modified even at scales much larger than ξ, in a well-controlled and rather universal manner.
We apply those results (i) to propose a refined interpretation of recent experiments on the motion of interfaces in liquid crystals and (ii) to describe the slow (so called 'creep') motion of interfaces in random media.

1D interfaces: a picture gallery

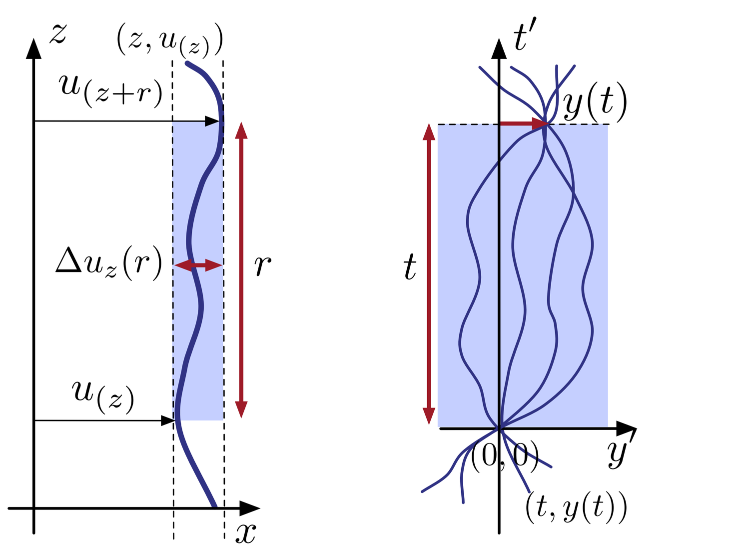


[Metaxas et al., APL **94** 132504 (2009)]

[Brittany]

[Courmayeur, 2011]

Interfaces in the Directed Polymer language



Geometrical parametrization:

- ⋆ longitudinal coordinate z
- ⋆ *univalued* transverse coordinate $u(z)$
- ⋆ no bubbles, no overhangs

Directed Polymer (DP) parametrization:

- ⋆ longitudinal coordinate: DP growing time $\boxed{t = z}$
- ⋆ transverse coordinate: DP endpoint $\boxed{y(t) = u(z)}$
- ⋆ working at fixed time t $\iff$ integration of fluctuations at scales smaller than t

Model & questions

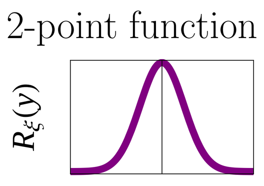
- Competing ingredients in the total energy $\mathcal{H}_V[y(t'), t]$:
  - ⋆ elastic energy (*flattens* the interface) **vs** disorder potential (*deforms* the interface)

$$\mathcal{H}^{\text{el}}[y(t'), t] = \frac{c}{2}\int_0^t dt' \left[\partial_{t'} y(t')\right]^2 \qquad \mathcal{H}_V^{\text{dis}}[y(t'), t] = \int_0^t dt'\, V\big(t', y(t')\big)$$

  - ⋆ no disordered potential $V(t,y)$: *diffusive* behaviour (typically, $y \sim t^{1/2}$), **Edwards-Wilkinson** (EW)
  - ⋆ disordered potential $V(t,y)$: *super-diffusive* behaviour (—, $y \sim t^{2/3}$), **Kardar-Parisi-Zhang** (KPZ)
- Nature of the disordered potential $V(t,y)$: "**Random-Bond**", *i.e.*

centered, Gaussian distributed, of 2-point function $\overline{V(t,y)V(t',y')} = D\delta(t'-t)R_\xi(y'-y)$

disorder correlator: smoothed delta



scaling as $R_\xi(y) = \frac{1}{\xi} R_{\xi=1}(y/\xi)$

- What is the distribution of the (quenched) polymer end-point **free-energy**, encoding its *fluctuations*?

partition function: $Z_V(t,y) = \int_{y(0)=0}^{y(t)=y} \mathcal{D}y(t') e^{-\frac{1}{T}\mathcal{H}_V[y(t'),t]}$ $\qquad$ free energy: $F_V(t,y) = -\frac{1}{T}\log Z_V(t,y)$

- What is the variance of the polymer endpoint at scale t?

encoded in the roughness $\quad B(t) = \overline{\langle y(t)^2 \rangle} = \overline{\dfrac{\int dy\, y^2 Z_V(t,y)}{\int dy\, Z_V(t,y)}}$

- Summary of parameters:

elastic constant c	disorder strength D	temperature T	disorder correlation length ξ

Evolution equations & Symmetries

- **Stochastic Heat Equation** for the partition function $Z_V(t,y)$

$$\partial_t Z_V = \Big[\frac{T}{2c}\partial_y^2 - \frac{1}{T}V(t,y)\Big] Z_V(t,y) \quad \textbf{(SHE)}$$

Linear, multiplicative noise, $Z_V(0,y) = \delta(y)$

- **Statistical Tilt Symmetry**:

$$\boxed{F_V(t,y) = \underbrace{c\frac{y^2}{2t} + \frac{T}{2}\log\frac{2\pi T t}{c}}_{\text{elastic contribution}} + \underbrace{\bar{F}_V(t,y)}_{\text{disorder contribution}}}$$

$\bar{F}_V(t,y)$ *invariant by translat° in distribution*

- Implies $B(t) = B_{\text{th}}(t) + B_{\text{dis}}(t)$ with

$$B_{\text{th}}(t) = \frac{Tt}{c} \quad \text{and} \quad B_{\text{dis}}(t) = \overline{\langle y(t) \rangle^2}$$

- **Kardar-Parisi-Zhang** equation for the free-energy $F_V(t,y)$

$$\partial_t F_V = \frac{T}{2c}\partial_y^2 F_V - \frac{1}{2c}\big[\partial_y F_V\big]^2 + V(t,y) \quad \textbf{(KPZ)}$$

Non-linear, additive noise,
$F_V(0,y)$: "sharp wedge" initial condition

- **Tilted** KPZ equation for the disorder free-energy $\bar{F}_V(t,y)$

$$\partial_t \bar{F}_V + \frac{y}{t}\partial_y \bar{F}_V = \frac{T}{2c}\partial_y^2 \bar{F}_V - \frac{1}{2c}\big[\partial_y \bar{F}_V\big]^2 + V(t,y)$$

Simple initial condition $\bar{F}_V(0,y) = 0$

Known results in the case of *uncorrelated* disorder ($\xi = 0$)

- **Central tools**: 2-point correlators $\qquad$ [related through $\bar{R}(t,y) = \frac{1}{2}\partial_y^2 \bar{C}(t,y)$]

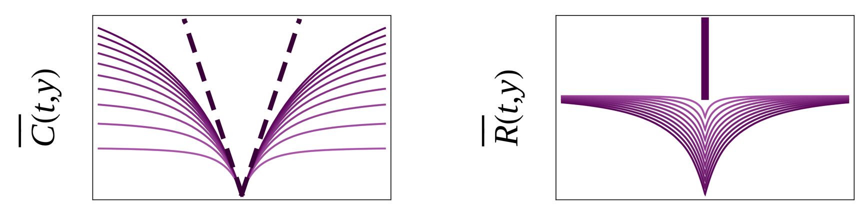
$$\bar{C}(t, y_2 - y_1) = \overline{(\bar{F}_V(t,y_2) - \bar{F}_V(t,y_1))^2} \qquad \bar{R}(t, y_2 - y_1) = \overline{\partial_y \bar{F}_V(t,y_1)\partial_y \bar{F}_V(t,y_2)}$$

- **Infinite time**: Brownian steady-state

$$\bar{C}(\infty, y) = \widetilde{D}|y| \qquad \bar{R}(\infty, y) = \widetilde{D}\delta(y) \qquad \boxed{\widetilde{D} = \frac{cD}{T}}$$

- **Large times**: Airy_2 scaling

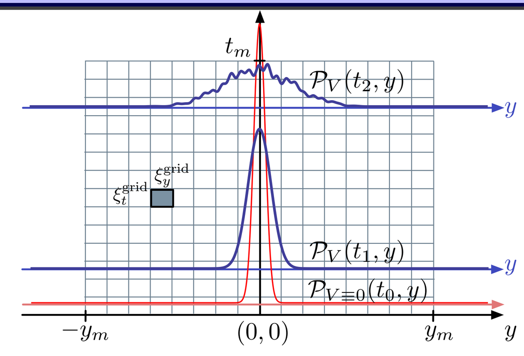
$$\bar{C}(t,y) = \widetilde{D}\sqrt{B_{\text{RM}}(t)}\, \hat{C}\big(y/\sqrt{B_{\text{RM}}(t)}\big)$$
$$\bar{R}(t,y) = \widetilde{D}/\sqrt{B_{\text{RM}}(t)}\, \hat{R}\big(y/\sqrt{B_{\text{RM}}(t)}\big)$$



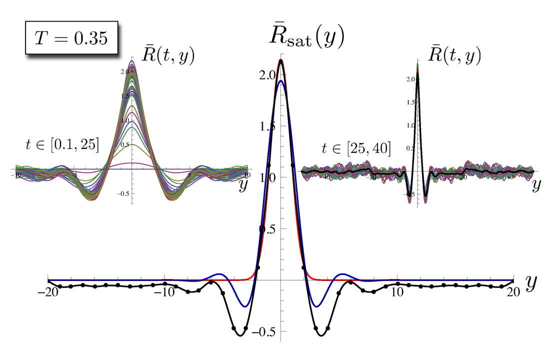
$$\boxed{B_{\text{RM}}(t) = \big[\widetilde{D}/c^2\big]^{2/3} t^{4/3}}$$

Numerical results for *correlated* disorder ($\xi > 0$)

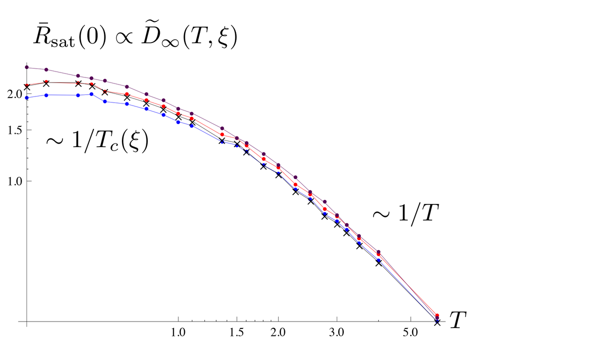
- **Numerical approach**
  - ⋆ Resolution of the *tilted* KPZ eq. for $\bar{F}_V(t,y)$
  - ⋆ Correlated disorder: *spline* of a disorder on a grid
  - ⋆ Initial condition at small t_0: parabola for $\bar{F}_V(t,y)$



- **Time-dependent and asymptotic** $\bar{R}(t,y)$



- **Amplitude** $\widetilde{D}_\infty$ of the free-energy fluctuations



Analytical results for *correlated* disorder ($\xi > 0$)

- **Large-time roughness, 2/3 exponent, link btw roughness and free-energy amplitudes**

$$B(t; c, \widetilde{D}, T, \xi) = \Big[\frac{\widetilde{D}}{c^2}\Big]^{\frac{2}{3}} t^{\frac{4}{3}} \times \overline{\frac{\int d\bar{y}\, \bar{y}^2 \exp\Big\{-\frac{1}{T}\big[\frac{\widetilde{D}^2}{c}t\big]^{\frac{1}{3}}\Big[\frac{\bar{y}^2}{2} + \bar{F}_1(1,\bar{y})\Big]\Big\}}{\int d\bar{y} \exp\Big\{-\frac{1}{T}\big[\frac{\widetilde{D}^2}{c}t\big]^{\frac{1}{3}}\Big[\frac{\bar{y}^2}{2} + \bar{F}_1(1,\bar{y})\Big]\Big\}}} \overset{\text{saddle } \bar{y}^\star \text{ at large } t}{\sim} \Big[\frac{\widetilde{D}}{c^2}\Big]^{\frac{2}{3}} t^{\frac{4}{3}} \overbrace{\overline{[\bar{y}^\star]^2}}^{t\text{-independent}}$$

- **High and low temperature regimes** $\boxed{T_{\mathbf{c}} = (\xi c D)^{1/3}}$ $\quad r_*(T) \equiv \frac{T^5}{cD^2}$, $\xi_{\text{th}}(T) \equiv \frac{T^3}{cD}$ $\quad [\xi_{\text{th}}(T_{\text{c}}) = \xi]$

$$B(t; c, D, T, \xi) \overset{\text{(exact)}}{=} \xi_{\text{th}}(T)^2 B\Big(\frac{t}{r_*(T)}; 1, 1, 1, \frac{\xi}{\xi_{\text{th}}(T)}\Big) \overset{(T \gg T_{\text{c}})}{\sim} \Big[\frac{\widetilde{D}}{c^2}\Big]^{\frac{2}{3}} t^{\frac{4}{3}} \quad \text{with} \quad \boxed{\widetilde{D}^{\text{high } T} = \frac{cD}{T}}$$

$$B(t; c, D, T, \xi) \overset{\text{(exact)}}{=} \xi^2 B\Big(\frac{t}{r_*(T_{\text{c}})}; 1, 1, \frac{T}{T_{\text{c}}}, 1\Big) \overset{(T \ll T_{\text{c}})}{\sim} \Big[\frac{\widetilde{D}}{c^2}\Big]^{\frac{2}{3}} t^{\frac{4}{3}} \quad \text{with} \quad \boxed{\widetilde{D}^{\text{low } T} = \frac{cD}{T_{\text{c}}}}$$

Experimental predictions

- **Interfaces in ferromagnetic materials**
  roughness amplitude A in $B(t) \overset{t\to\infty}{\sim} A\, t^{4/3}$

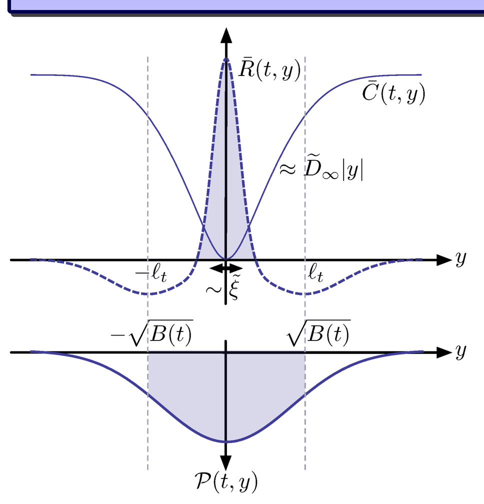
$$A \overset{(T \gg T_{\text{c}})}{\sim} \Big(\frac{D}{cT}\Big)^{\frac{2}{3}} \qquad A \overset{(T \ll T_{\text{c}})}{\sim} \Big(\frac{D^2}{c^4 \xi}\Big)^{\frac{2}{9}}$$

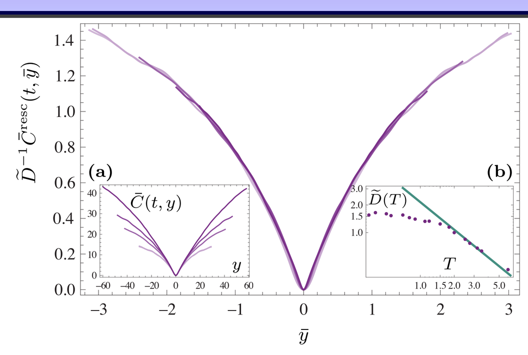
- **Interfaces** $\boxed{h(t,y) \cong F(t,y)}$ **in liquid crystals**

$$\partial_t h = v_\infty + \nu\partial_y^2 h + \frac{\lambda}{2}\big[\partial_y h\big]^2 + V$$

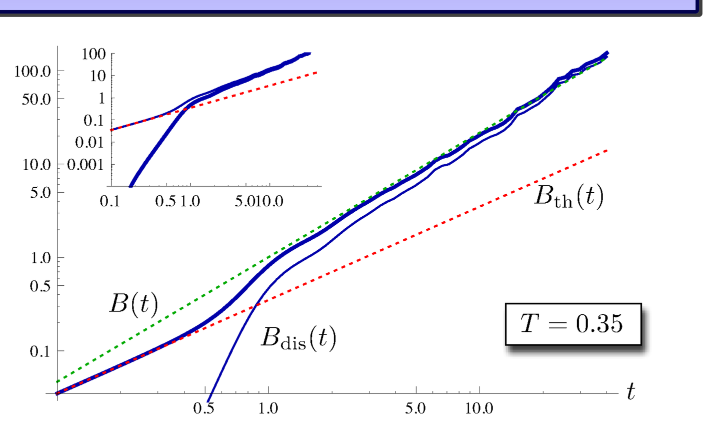
$$\big[A \cong \Gamma^{\frac{2}{3}}\big] \qquad \Gamma \overset{(\lambda \ll \lambda_{\text{c}})}{\sim} \frac{D^2\lambda}{\nu^2} \qquad \Gamma \overset{(\lambda \gg \lambda_{\text{c}})}{\sim} \frac{D^{\frac{4}{3}}}{\lambda^{\frac{1}{3}}\xi^{\frac{2}{3}}}$$

Summary





$$\bar{C}^{\text{resc}}(t,\bar{y}) = \frac{\bar{C}\big(t, y/\sqrt{B(t)}\big)}{\sqrt{B(t)}}$$



Scale-dependent roughness

Open questions

Determine at $\xi > 0$:

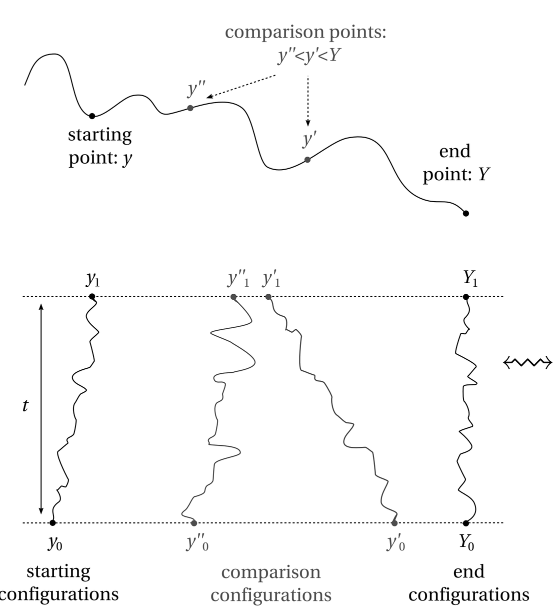
- ⋆ full temperature crossover
- ⋆ link between $R_\xi(y)$ and $\bar{R}(\infty, y)$

Translation for:

- ⋆ quantum bosons (cf. replicæ)
- ⋆ generalized Airy process

Experiments:

- ⋆ a regime where ξ matters?
- ⋆ scaling of correlators $\bar{C}$, $\bar{R}$



Creep law: (**non-linear**) mean velocity with a small force f

$$v(f) \sim e^{-(f_{\text{c}}/f)^{1/4}}$$

Usual derivations: from equilibrium laws / in high dimensions (Functional Renormalization Group in $d = 4 - \varepsilon$, $\varepsilon \ll 1$)
⇝ Derivation from the DP: *a priori* wrong demonstration (analogy with a single particle dynamics) but correct result, with ξ-dependent

$$f_{\text{c}} = \frac{\widetilde{D}^7}{c^5 D^4}$$

References:

- E Agoritsas, V Lecomte & T Giamarchi, `arXiv:1209.0567`
- E Agoritsas, S Bustingorry, V Lecomte, G Schehr & T Giamarchi, Phys Rev E **86** 031144 (2012)
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