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Creep in one dimension and phenomenological theory of glass dynamics

Pierre Le Doussal [a,c,*,1], Valerii M. Vinokur [b,c]

[a] *CNRS-Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, 24 rue Lhomond, F-75231 Paris, France*
[b] *Materials Science Division, Argonne National Laboratory, Argonne IL 60439, USA*
[c] *ITP, University of California, Santa Barbara CA 93106, USA*

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Abstract

The dynamics of a glass transition is discussed in terms of the motion of a particle in a one-dimensional correlated random potential. An exact calculation of the velocity V under an applied force f demonstrates a variety of dynamic regimes depending on the range of correlations. In a gaussian potential with correlator $C(x) = x^\gamma$, we find a transition from ohmic behavior ($\gamma < 0$) to creep motion $V \sim \exp(-\text{const}/f^\mu)$ ($0 < \gamma < 1$). This provides a generic picture of the glass transition in systems where long-range correlations in the effective disorder develop due to elasticity such as elastic manifolds subject to quenched disorder and the vortex-glass transition in superconductors.

The driven dynamics of physical systems which can be modelled as elastic manifolds in quenched random media has received a lot of attention recently. Prominent examples of such systems are the roughening of domain walls [1], directed polymer growth [2], motion of dislocations in disordered media [3], dynamics of charge density waves [4], and surface growth in a random environment [5]. This recent interest was partly motivated by extensive studies of vortex dynamics in high-T_c superconductors. It was shown that this problem is related to the dynamics of an elastic manifold subject to quenched disorder [6–9], and a significant progress in *qualitative* understanding was achieved. The motion is viewed as a sequence of thermally activated jumps of the optimal "cell" of the manifold from one metastable state to the next as favored by the applied force. In most of these systems the activation barriers for such a motion, which we will refer to as creep motion, depends on the applied force f and diverges as $f \to 0$, giving rise to a strongly non-linear velocity versus applied force dependence. The unlimited growth of the creep barriers is taken now as the characteristic feature and operational definition of glassy dynamics. Usually [8] the barriers grow as $U_B(f) \sim f^{-\mu}$ leading to a typical velocity versus applied force dependence (or I–V curve) of the form $V = e^{-1/(Tf^\mu)}$ in the creep regime.

While there is now a consistent qualitative picture of the low-temperature creep motion [10,6], it is based mostly on scaling arguments and numerical simulations, and a rigorous analysis is still lacking. Although interesting new results have been obtained recently for the non-equilibrium dynamics of mean-field models of glasses [11] and for particle dynam-

* Corresponding author.

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ics in an infinite-dimensional random potential [12,13], a general analytical derivation in physical models remains an unsolved problem. In the absence of a rigorous analysis of realistic physical situations one is seeking for models which are simple enough to be treated analytically and yet are able to mimic the large diversity of dynamics of real glassy systems. A well known example is the problem of a single particle driven by an external force f and subject to a one-dimensional random force field with gaussian short-range correlations. The term random force means that the correlator of the random potential $U(x)$ is $\overline{(U(x)-U(y))^2} \sim \Delta |x-y|$ where Δ characterizes the strength of the random potential. This model is known as Sinai's model and has long been a subject of extensive studies [15–21,3] The remarkable result obtained for this model is that even at finite temperature the mobility vanishes below a threshold force $f_{th} \sim \Delta/2T$. Moreover this system was found to exhibit anomalous diffusion, and aging phenomena [20] very much like what is observed in spin glasses [22,11].

In this letter we study a wider class of one-dimensional random potentials with arbitrary correlations and find that our model can reproduce most of the existing regimes of glassy dynamics. The letter is organized as follows. First we derive an exact formula for the velocity in a one-dimensional medium. The result is physically transparent and simple enough to allow for analytical treatment of very general potentials. Then we study gaussian random potentials with a general correlator such that at large distances $|x-y| \gg a$:

$$\overline{(U(x)-U(y))^2} \equiv K(x-y) \sim \Delta |x-y|^{\gamma}, \quad (1)$$

and in Fourier space $\tilde{K}(q) = \overline{U(q)U(-q)} \sim_{q\to 0} q^{-(1+\gamma)}$ with $K(z) = 2\int_{-\Lambda}^{\Lambda}(\mathrm{d}q/2\pi) \times(1-\cos qz)\,\tilde{K}(q)$. We find that there are several regimes depending on how correlated the potential is. If correlations are short range ($\gamma<0$), we recover a "viscous flow" (ohmic) regime where a linear response $V \sim f$ holds for small f. If correlations grow with $0<\gamma<1$ we find a new creep regime, $V \sim \exp(-1/f^{\mu})$. The case $\gamma=0$ corresponds to the transition between these two regimes and we find a critical power-law V versus f dependence, reminiscent of the vortex-glass transition behavior [7]. Finally $\gamma=1$ corresponds to Sinai's case where $V=0$ below a threshold force.

This suggests a deep connection with the motion of elastic manifold in a random medium (such as vortex systems in type-II superconductors). In these systems creep behavior arises from the interplay of the elasticity and pinning potential [6] which both determine the creep barriers U_b. The bare pinning potential is uncorrelated but the elasticity of the manifold generates long-range correlations in the effective potential U_b that determines the motion of the manifold. The role of elasticity as a tuning mechanism for correlations becomes transparent upon noticing that the formation of the vortex glass is caused by a drastic change in elastic properties. Namely, the onset of shear modulus at the freezing point develops long-range correlations in the vortex system [6]. A phenomenological approach to describe a generic glass transition is to introduce a correlation length ξ_G characterizing the spatial range of critical correlations, which diverges at the transition. The choice of $K(x)$ on both sides of the transition is dictated by physical considerations. In the correlated phase (i.e. glass phase) the natural choice is a gaussian random potential with correlator of the form

$$K(x-y) \sim \Delta\left(\left(\frac{|x-y|}{\xi_G}\right)^{\gamma} + \log\left(\frac{|x-y|}{a}\right)\right). \quad (2)$$

The first term which dominates at large distance describes long-range correlations in the random potential and generalizes Sinai's model. The second term describes the behavior at the critical point $\xi_G=\infty$. Indeed one has $\gamma=0$ and $\tilde{K}(q) \sim 1/q$ at the transition and thus $K(x) \sim \Delta \ln x$. The form (2) is an interpolation resulting from the crossover between the critical fixed point and the fixed point describing the glass phase. In the uncorrelated phase correlations are short range and one chooses a correlator as $\tilde{K}(q) = 1/(q^2+(\xi_G)^{-2})^{1/2}$, i.e. $K(x) = K_0(x/\xi)$, so as to reproduce the critical behavior for $a \ll x \ll \xi_G$. An identical scenario was demonstrated using RG for the correlations in the free-energy landscape at the glass transition in surface growth models, such as the directed polymer in $d \geqslant 2+1$ [14]

On a mathematical level the present model is the $d = 0$, $n = 1$ version of the problem of the dynamics of manifolds of internal dimension d, in a n dimensional space. Remarkably, the case $d = 0$ and $n \to \infty$ was recently studied by completely different techniques and seems to exhibit similar regimes [12].

We consider the Langevin diffusion in the one-dimensional quenched random potential $U(x)$ in the presence of a global bias f and thermal white noise $\eta(t)$:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -\nabla U(x(t)) + f + \eta(t), \tag{3}$$

with $\langle \eta(t)\eta(t')\rangle = 2T\delta(t-t')$ and T is the temperature. The probability density $P(x, t)$ and the current $J(x, t)$ satisfy

$$\frac{\partial P(x,t)}{\partial t} = -\nabla J(x,t), \tag{4}$$

with $J(x, t) = -T\nabla P(x, t) + (f - \nabla U(x)) \times P(x, t)$.

To derive the analytic expression for the velocity V we generalize to continuum models the method introduced by Derrida [18] for discrete hopping problems. We consider an infinite periodic environment, i.e. a periodic random force $\nabla U(x)$, of period L. The limit $L \to \infty$ is taken at the end [23]. One defines the periodized probability $\tilde{P}(x) = \Sigma_k P(x + kL)$ which obeys the same equation (4) as P, and corresponds to diffusion on a periodic ring of size L. Using Eq. (4) the velocity for the particle on the infinite line can be expressed as

$$\frac{\mathrm{d}\langle x(t)\rangle}{\mathrm{d}t} = -\int_{-\infty}^{+\infty} \mathrm{d}x x\nabla J = \int_{-\infty}^{+\infty} \mathrm{d}x J(x,t) = \int_0^L \mathrm{d}x \tilde{J}(x,t), \tag{5}$$

where $\tilde{J}(x, t) = -T\nabla \tilde{P}(x, t) + (f - \nabla U(x)) \times \tilde{P}(x, t)$. At long time $\tilde{J}(x, t)$ goes to a constant $\tilde{J}$ and the asymptotic velocity V is exactly given by $V = \tilde{J}L$. To find $\tilde{J}$ for a fixed L and disorder configuration one must solve the stationarity equation:

$$T\frac{\partial \tilde{P}(x)}{\partial x} + (\nabla U(x) - f)\tilde{P}(x) = -\tilde{J}, \tag{6}$$

with the two additional conditions $\tilde{P}(0) = \tilde{P}(L)$ and $\int_0^L \mathrm{d}x \tilde{P}(x) = 1$. The stationary solution with zero current $\tilde{J} = 0$, i.e. the Gibbs distribution $P_0(x) = \exp((1/T)(-U(x) + fx))$ does not, in general, satisfy the periodic boundary conditions: Thus V can be found from the solution with non-zero current:

$$\tilde{P}(x) = \frac{\tilde{J}}{T}\left(\frac{\int_0^L \mathrm{d}y\, \mathrm{e}^{(U(y)-U(x)+f(x-y))/T}}{1 - \mathrm{e}^{(U(L)-U(0)-fL)/T}} - \int_0^x \mathrm{d}y\, \mathrm{e}^{(U(y)-U(x)+f(x-y))/T}\right); \tag{7}$$

$\tilde{J}$ and thus V follow from the normalization condition for $\tilde{P}$. In the limit $L \to \infty$, imposing the restriction $U(0) = U(L)$, unimportant for $f > 0$, Eq. (7) simplifies to

$$\tilde{P}(x) = \frac{\tilde{J}}{T}\int_0^{+\infty} \mathrm{d}z\, \mathrm{e}^{(U(x+z)-U(x)-fz)/T} \tag{8}$$

and one gets the general formula for V:

$$\frac{1}{V} = \frac{1}{T}\int_0^{+\infty} \mathrm{d}z\, \mathrm{e}^{-fz/T}\langle \mathrm{e}^{(U(x+z)-U(x))/T}\rangle_x \tag{9}$$

valid for an arbitrary potential $U(x)$. $\langle A\rangle_x$ denotes the translational average $\langle A\rangle_x = \lim_{L\to\infty} L^{-1} \times \int_0^L \mathrm{d}x A(x)$. The average in Eq. (9) exists quite generally and is independent of the configuration of the random potential, i.e. the velocity is self-averaging. The physical interpretation of Eq. (9) in terms of an Arrhenius waiting time is transparent. The average waiting time $1/V$ is a sum of Boltzmann weights associated with the barriers the particle must overcome to move in the direction of the driving force. The highest barriers $U(x+z) - U(x)$ with $z > 0$, produce the largest waiting times.

The expression (9) reveals immediately several general features. At large f one has $V \approx f$. At small force $f \to 0$, the response will be linear only if the barriers saturate, i.e do not grow at large distance. If the potential is uncorrelated at large distances such that $\langle \mathrm{e}^{(U(x+z)-U(x))/T}\rangle \to \langle \mathrm{e}^{U/T}\rangle\langle \mathrm{e}^{-U/T}\rangle$ when $z \to \infty$, then

$$V \alpha \frac{D}{D_0} f = \frac{f}{\langle \mathrm{e}^{U/T}\rangle\langle \mathrm{e}^{-U/T}\rangle}, \tag{10}$$

where D and D_0 are the diffusion coefficients in the presence and in absence of disorder, respectively and the Einstein relation holds. When $D \ll D_0$ the V–f curve will show strong nonlinearity at intermediate scales where the transition between the low-force and high-force regime of motion occurs.

We now turn to a detailed study of gaussian disorder with correlator $K(x)$. Upon averaging over disorder Eq. (9) yields

$$\frac{1}{V} = \frac{1}{T}\int_0^{\infty} \mathrm{d}x \exp\left(-\frac{fx}{T} + \frac{K(x)}{2T^2}\right). \tag{11}$$

The choice of $K(x)$ as in Eq. (2) gives rise to several regimes of particle dynamics depending on the range of the correlations of the random potential.

First we consider *Sinai's case*, $\gamma = 1$. For $\gamma > 1$ the integral in Eq. (11) *diverges* and the velocity is zero. Sinai's model corresponds to $\gamma = 1$ and appears as a marginal case where the integral (11) diverges for $f < f_{\mathrm{th}} = \Delta/(2T\xi_G)$ and $V = 0$ while $V = f - f_{\mathrm{th}}$ exactly for $f > f_{\mathrm{th}}$, in agreement with previous results [15–17,19,20]. The system with $\gamma = 1$ exhibits algebraic distributions of waiting times which gives rise to aging phenomena [20]. Therefore Sinai's model mimics essential aspects of the spin-glass behavior. Interestingly, this V versus f dependence mimics also the dry-friction phenomenon.

Next we consider *creep motion*, $0 < \gamma < 1$. In the intermediate case $0 < \gamma < 1$ one finds the "creep" dynamics regime. Defining the dynamical exponent $z = 2 + (\Delta/2T^2)$ and the characteristic force $f_c = (\Delta/2T^2)^{1/\gamma}\, T/\xi_G$ one arrives at

$$\frac{T}{V} = \frac{1}{f_c^{z-1}} H(f/f_c)$$

$$H(y) = \int_0^{\infty} \mathrm{d}v\, v^{z-2} \exp(-yv + v^{\gamma}). \tag{12}$$

Note that f_c reduces to the threshold force f_{th} when $\gamma \to 1$. At $\gamma < 1$ the sharp threshold disappears but at $f \ll f_c$ the V versus f dependence shows a strongly nonlinear behavior with an essential singularity at small f. Using the steepest-descent method at $f \ll f_c$ one finds

$$V = ATf_c^{z-1}\left(\frac{f}{f_c}\right)^{\frac{z-1-(\gamma/2)}{1-\gamma}} \exp\left(-(1-\gamma)\left(\frac{\gamma f_c}{f}\right)^{\mu}\right),$$

$$\mu = \frac{\gamma}{1-\gamma}, \tag{13}$$

with $A \sim \sqrt{\gamma(1-\gamma)/2\pi}$. The exponential factor holds for any correlator behaving as a power law $\sim x^{\gamma}$ at large distances. The preexponential factor depends on details of the crossover of the correlator to the logarithmic regime ($x \ll \xi_G$). In this creep regime the linear response at $f \to 0$ is absent and the characteristic barriers which control the dynamics diverge as $1/f^{\mu}$.

Further, the *critical* case. Critical behavior, which generically corresponds to $\gamma = 0$, can equivalently be achieved by taking $\xi_G = \infty$ in Eq. (2). The V versus f characteristics becomes a power law at small force. In that case we have $K(x) \sim \Delta \log|x|$ at large x and Eq. (11) gives

$$V = \frac{f^{z-1}}{\Gamma(z-1)T^{2+z}}, \tag{14}$$

with $z = 2 + (\Delta/2T^2)$, resembling the power-law critical behavior proposed at the vortex-glass transition. This transition separates the creep dynamics $V \sim \exp(-\mathrm{const}/f^{\mu})$ in the vortex-glass state from the ohmic behavior $V \propto f$ above the transition. Indeed one finds that the scaling function $H(y)$ of Eq. (12) reproduces the correct scaling behavior $H(y) \sim y^{1-z}$ in the critical region $y \gg 1$ which leads to Eq. (14) for $T/a^2 \gg f \gg f_c$. Using Eq. (10) we find the critical behavior $D \propto \xi_G^{2-z}$. Since in the vortex system the voltage is proportional to the vortex velocity and force is proportional to applied current, the obtained behavior of D reproduces the critical behavior of the resistivity ρ at the transition, ξ_G playing the role of the vortex-glass critical length.

Last, we have *ohmic behavior*, $\gamma \leq 0$. For short-range correlations $\gamma < 0$, $\overline{U(x)^2}$ is finite and one recovers the Einstein relation (10) with $D > 0$ and the linear response. Let us analyze in detail the regime of ohmic motion, corresponding to a finite correlation length. We denote $\overline{U(x)U(0)} = \Delta C(x)$, with $C(0) = 1$ and $C(x)$ decays to zero on a length scale ξ.

$$\frac{1}{V} = \frac{1}{T}\int_0^{\infty} \mathrm{d}z \exp\left(-\frac{fz}{T} + \frac{\Delta}{T^2}(1 - C(z))\right). \tag{15}$$

Two ohmic regimes, one at small force with $V \propto \mathrm{e}^{-\Delta/T^2} f$ and one at large force with $V \propto f$, appear. At a high temperature these two regimes match smoothly. At low temperature there is a sharp crossover. A depinning temperature $T_p = \sqrt{\Delta}$ separates these two different behaviors which we call "unpinned" and "pinned" respectively. For $T \gg T_p$ the particle motion is mostly unpinned and the smooth crossover occurs at $f_c \sim T/\xi$. For $T \ll T_p$ a new characteristic force f_p arises which marks the crossover between $V \propto e^{-\Delta/T^2} f$ for $f \ll f_p$ and $V \propto f$ for $f \gg f_p$. To estimate f_p we use $C(z) = \exp(-(z/\xi)^2)$ which leads to $f_p \approx (T/\xi)\sqrt{\Delta/T^2} \sim \sqrt{\Delta}/\xi$. Note that f_p does not depend on temperature and has thus a limit when $T \to 0$. At $T \to 0$ the curve V versus f becomes $V = \theta(f - f_p)(f - f_p)$. It is interesting to mention that this behavior can be obtained correctly using the perturbation theory analogous to that used for calculation of critical currents in type-II superconductors [6]. Disorder-induced corrections to the velocity are given by $\delta v/v = \Delta \xi v T^{-3}((T/\xi v) - \arctan(T/\xi v))$. Equating $\delta v/v \approx 1$ we recover the above result for f_p at low temperature $T \ll T_p$ while for $T \gg T_p$, $\delta v/v$ never reaches 1 and thus f_p does not exist. This analysis assume a finite mean squared force $\langle (\nabla U)^2 \rangle \sim f_p^2 < \infty$, i.e. that the random potential is smooth. If we choose instead $C(z) = \exp(-|z/\xi|)$ an exact calculation from Eq. (15) gives

$$V = \frac{T}{\xi \gamma(f\xi/T, 1)} \left(\frac{\Delta}{2T^2} \right)^{f\xi/T} \exp(-\Delta/2T),$$

where γ is the incomplete Gamma function. In that case the average force is infinite and at $T = 0$ the particle remains pinned at any applied force.

The above model may apply directly to a string (directed polymer) of length L in d dimensions driven by its tip at $z = 0$ over point-like impurities. An example is a flux line in a superconductor in the presence of an external current J. Because of screening at small fields the external Lorentz force of modulus $f \propto J$ is exerted only on the end of the line, over a length of order λ. Let $F(r)$ be the free energy corresponding to equilibrium positions r of the tip at $z = 0$ with the other end $z = -L$ fixed at $r = 0$ for large L. One can view the tip as a particle driven in the rough potential $F(r)$ with $\overline{(F(r) - F(0))^2} \sim r^\gamma$ and [1,2] $\gamma = 2\theta/\zeta$. Moving the tip by r provokes a reorganization of the string up to length $z = r^{1/\zeta}$ away from the tip. It thus leads to correlated barriers and to the V–f characteristics found above. Since $\zeta \approx 6/(8 + d)$ and $\theta = 2\zeta - 1$ one has $\gamma \approx (4 - d)/3$. Thus $\gamma = 1$ in $d = 1 + 1$, an exact result, and $\gamma \approx \frac{2}{3}$ in $d = 2 + 1$. If this picture is qualitatively correct a flux line driven by its tip in $d = 1 + 1$ would experience a threshold. Below f_c the motion of the tip would be as $r \sim t^{f/f_c}$ and $V = 0$. By analogy with Sinai's model [20], we also predict aging effects in this system. One has $V \sim \exp(-1/f^2)$ in $d = 2 + 1$. Of course the true motion is more complicated and the string might not be assumed to be in equilibrium.

Note that in one-dimensional motion the velocity can be dominated by rare large barriers, as is the case for the exponent μ in Eq. (13), whereas diffusion at zero force $f = 0$, given [24,25] by $x \sim (\log t)^{2/\gamma}$, is dictated by *typical barrier* $E_b \sim x^{\gamma/2}$. In higher-dimensional space, such as the configurational space of a string, there are many paths in parallel from one point to another which may allow the avoidance of atypically high barriers. This could effectively cut the tails of the distribution of barriers, and the gaussian assumption may not be justified. This effect can be accounted for within our one-dimensional model by allowing for more general distributions for barriers. Averaging Eq. (9) over disorder, one can write

$$\frac{1}{V} = \frac{1}{T} \int_0^\infty dz \int dE_b P(E_b, z) \exp((E_b - fz)/T), \tag{16}$$

where $P(E_b, z)$ is the probability of a barrier $E_b = U(x + z) - U(x)$ corresponding to separation z. The previous calculation corresponds to a gaussian $P(E_b, z) \sim \exp(-E_b^2/2K(z))$. A more general form is

$$P(E_b, z) = z^{-\gamma/2} Q(E_b/z^{\gamma/2}). \tag{17}$$

The behavior at small f is governed by the large barriers. Thus we suppose that $Q(u) \sim \exp(-u^\delta/2\Delta)$ at large u. Then using the saddle-point method one obtains

$$V \sim \exp(-CT^{-(1+\nu)} f^{-\mu}),$$
$$\mu^{-1} = \frac{2(\delta - 1)}{\gamma\delta} - 1, \tag{18}$$

with $\nu = 1/(\delta - 1 - \gamma\delta/2)$ and $C = \nu^{-1}(2/\gamma\delta)^{1+\nu}(2\Delta)^{-1/\nu}$. This is the most general case containing the gaussian case for $\delta = 2$. For

$\delta \to \infty$, i.e. when the distribution has no tails, the typical barriers dominate and $\mu = \mu_{typ} = \gamma/(2-\gamma)$. This calculation shows, however, that even for rapidly falling tails, δ large but finite, the exponent μ can be different from the value μ_{typ} obtained by considering typical barriers.

In conclusion, we found stretched exponential velocity versus force characteristics, $V \sim \exp(-\text{const}/f^{\mu})$, in a simple model of a particle diffusing in a 1D random potential with correlated barriers. There is a transition between this creep motion and ohmic motion, characterized by a power-law critical behavior at the transition. This transition occurs as the correlations in the random potential change from short range to long range. We stressed the physical analogies with the creep motion of driven flux lines in a glassy state, also characterized by correlated barriers. The transition found here is similar to the vortex-glass transition which can thus be viewed as a transition from the short-range (in the liquid) to the long-range correlated behavior (in a glassy state) of the effective potential seen by a vortex configuration. The above results also establish a connection between spin and vortex glasses. Our model exhibits both creep behavior specific to vortex glasses and aging, which is recognized as the essential characteristic of spin glass dynamics, upon tuning the degree of correlation of random field.

Note added in proof

After completion of this work we received a preprint by S. Scheidl where Eq. (9) is also derived in a different context.

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